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Research Article

MATHEMATICS

Adomian decomposition method for time fractional diffusion-reaction equation

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ABSTRACT

In this paper, the Adomian decomposition method (ADM) and specially its symbolic capability on the usage for determination of a solution to partial differential equations is explored. The Adomian decomposition method is used to solve the time fractional diffusion-reaction equation. This method is applied for two models of the time fractional diffusion-reaction equations (Autocatalytic reaction and Bistable and Schlogl models). The results given by this method show that, the effectiveness of the solution is closed Adomian decomposition method for time fractional diffusion-reaction equation to practical proposals of our research and is close to the exact solution. Moreover, the current method is introduced for the solution of the time fractional diffusion-reaction equation. The concentration of the species function is proportional directly with all the physical parameters in the “Bistable or Schlogl model” and inversely in case of” Autocatalytic model” respectively. The approximate solution considered that, the first and second driven terms of the decomposition are enough for the approximate solution.

Introduction

Since Adomian invented Adomian decomposition method (ADM) in early 1980s and wrote two masterpieces by **(Duan et al., 2012)** for it. ADM has become a very popular technique. It has been extensively used to solve many types of problems, including PDEs, ordinary differential equations (ODEs), functional equations, etc. Normally we can get an approximate solution of truncated series by ADM. Moreover, for a certain problem it even becomes an analytic method to get analytic solution of infinite series, if we could find the pattern of series coefficients. Time fractional diffusion-reaction equation is a fundamental partial differential equation (PDEs) in fluid mechanics. Analytical solutions of PDEs modeling physical phenomena exist in some cases. ADM represents a powerful technique for solving all kinds of ordinary and partial differential equations. It is valid and effective for linear and nonlinear equations. Its efficiency is well known to possess rapidly convergent solutions and only a few iterations reach the desired accuracy. Although first order PDEs are relatively simple compared with higher order ones. The fractional differential equations have been the focus of many researchers because of their frequent appearance in biology, physics, engineering, viscoelasticity and fluid mechanics as given by authors **(Mohammadein et. al., 2020; El-Amin, 2025-d)**.

A survey of the ADM and its applications to fractional differential equations was presented by **(Duan et al., 2012)**. **Hemeda et al., 2021** study the iterative solution of the fractional fluid flow between two infinite parallel plates. Similarly, the ADM is employed to solve the time fractional Burgers equation as provided by **(Akpan, 2015; Alwehebi, et al., 2023)**. The Adomian decomposition method was examined by **(Tzon et al., 2021)** for first-order PDEs involving unprescribed data. **(Fuad et al., 2022)** address the electro-magneto hydrodynamic fractional-order fluid problems using new similarity transformations. The spatial fractional boundary-layer flow over a vertical plate approximately is investigated by **(Mohammadein et. al., 2020)**.

Gul et al., 2021 gave similarity variable transform from PDEs into fractional-order ODEs by special substitution.

Recently, the modeling of fractional transport phenomena in fluid flow is investigated by **(El-Amin, 2025-d)** with wide applications in industry and medical sciences. Moreover, some methods for solving fractional PDEs are studied for renewable energy systems as given by **(El-Amin 2024-a, b, and c)**. Also, methods in an optimal control efforts for solving fractional differential equations (FDEs) are applied in different fields **(Ali et al., 2024 and El-Mesady et al., 2025)**. In normal partial differential equations, there are many problems of fluid flow and bubble growth are studied by **(Abo-Eldahab, 2004; Abu-Nab et al. 2022)**.

Adomian Decomposition Method

A solution of nonlinear partial differential equation is illustrated by Adomian decomposition method in the general form:

$$L^t\left[z(\mathrm{y},t)\right]+R\left[z(\mathrm{y},t)\right]+N\left[z(\mathrm{y},t)\right]=f(\mathrm{y},t) \qquad (1)$$

where $L^t[.]=\frac{\partial}{\partial t}[.]$ show the partial derivative operator with regard to time t. Although R[.] is a linear operator, partial derivatives with respect to y are typically taken into consideration. Additionally, N[.] represents a nonlinear analytic operator and a non-homogeneous arbitrary function, $f\left(\text{y},t\right)$ is assumed to be independent of function of z(y, t). The function z(y,0) = g(y) represents the initial condition of Eq. (1), and it considered together with the initial condition z(y,0) = g(y). In the following, the operator L^t is invertible is assumed, and therefore we can apply L^{-1} to both sides of Eq. (1), then

$$\begin{aligned} z\left(\text{y},t\right) &= g\left(y\right)-L^{-1}R\left[z\left(\text{y},t\right)\right] \\ &-L^{-1}N\left[z\left(\text{y},t\right)\right]+L^{-1}\left[f\left(\text{y},t\right)\right] \end{aligned} \tag{2}$$

The existence of a series solution of Eq. (1) is assumed in which z(y,t) can be represented by

$$z\left(\text{y},t\right)=\sum_{n=0}^{\infty} z_n\left(\text{y},t\right) \tag{3}$$

Moreover, the nonlinear term N[z(y, t)] can be decomposed according to

$$N\left[z\left(\text{y},t\right)\right]=\sum_{n=0}^{\infty} A_n\left(,t\right) \tag{4}$$

where {An} $0\le n\le\infty$ are indicated to the Adomian polynomials. They can be calculated on basis of the simple rule

$$A_n\left(z_0,z_1,\dots,z_n\right)=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\left(\sum_{n=0}^{\infty} q_n\lambda^n\right)^m\right]_{\lambda=0} \tag{5}$$

By substituting the series expansions (3) and (4) into Eq. (1), then

$$\begin{aligned} \sum_{n=0}^{\infty} y_n\left(x,t\right) &= g\left(x\right)-L^{-1}R\left[\sum_{n=0}^{\infty} y_n\left(x,t\right)\right] \\ &-L^{-1}\sum_{n=0}^{\infty} A_n\left(x,t\right)+L^{-1}\left[f\left(x,t\right)\right] \end{aligned} \tag{6}$$

In this paper, the problem of time fractional diffusion-reaction equation is solved by using ADM for a given two models. The Autocatalytic represents the first model. The second model introduced as a Bistable and Schlogl model. Both models are studied under the effect of fractional α diffusion coefficient, power parameter “m”, and the time “t”. The only first and second driven terms of the decomposition is considered for the approximate solution.

Analysis

The diffusion equation under the effect of rate of chemical reactions is considered as a partial differential equation. The general form of distribution for a concentration of the species has the form:

$$\frac{\partial^\alpha q}{\partial t^\alpha}=d_f\nabla^2 q+R \tag{7}$$

where:

$q=q\left(x,y,z,t\right)$ is the concentration of the species.

t is time.

x,y,z represents the spatial variables.

d_f is the diffusion coefficient.

∇^2 is the Laplacian operator, representing diffusion.

$R=R\left(\text{q},x,y,z,t\right)$ is the reaction term, describing the rate of chemical reactions.

Rate of chemical reaction

The specific forms for $R = R(q)$ as a function of the concentration are studied in the two following cases:

Autocatalytic Reactions:

$$R = kq^m \quad . \tag{8}$$

The above equation (8) represents a reaction rate, which are a function of concentration of the reactant raised to some power “m”

Bistable or Schlogl Model:

A model that can lead to bi-stability in the system.

$$R(q) = k_1 - k_2 q + k_3 q^2 - k_2 q^3 \quad . \tag{9}$$

Adomian Decomposition Method for Autocatalytic Reactions

The linear fractional heat reaction-equation and the specified initial condition are considered as follows:

$$\frac{\partial^\alpha q}{\partial t^\alpha} = \sum_{i=1}^{n} d_f \frac{\partial^2 q_i}{\partial x_i^2} + kq^m \quad ,$$

$$q(x,y,z,0) = f(x,y,z) \quad , \tag{10}$$

for the above equation (10), applying $D_t^{-\alpha}$, then:

$$q - q_0 = D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_i}{\partial x_i^2} + kq^m\right) \tag{11}$$

The concentration of the species in terms of Adomian decomposition becomes:

$$q = \sum_0^\infty q_j$$

$$D_t^{-\alpha}\left(kq^m\right) = \sum_0^\infty A_j \tag{12}$$

where A_j are Adomian polynomials that depend on $q_0, q_1, q_2, \ldots\ldots\ldots q_j$. The resulting Expression becomes:

$$\sum_0^\infty q_j = f + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 \sum_0^\infty q_j}{\partial x_i^2}\right) + D_t^{-\alpha}\left(\sum_0^\infty A_j\right) \tag{13}$$

The recurrence relation is given by:

$$q_0 = f$$

$$q_{j+1} = q_j + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^2}\right) + D_t^{-\alpha} A_j \tag{14}$$

The Adomian polynomial is determined by introducing a parameter” λ” as follows:

$$A_i = \frac{k}{\lambda_j}\left[\sum_0^\infty f_m q_j \lambda^j\right]_{\lambda=0} \tag{15}$$

Then in view of equations (14) and (15), we have:

$$q_1 - q_0 = D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^2}\right) + D_t^{-\alpha} A_j \tag{16}$$

By applying the ADM we can write

$$q_{j+1} = D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^2}\right) + D_t^{-\alpha} A_j \tag{17}$$

so that

$$A_i = \frac{k}{j!}\frac{d^j}{d\lambda^j}\left[\left(\sum_0^\infty q_j \lambda^j\right)^m\right]_{\lambda=0} \tag{18}$$

$$\begin{aligned} A_0 &= \frac{k}{0!}\frac{d^0}{d\lambda^0}\left[\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^m\right]_{\lambda=0} \\ &= kq_0{}^m = kf^m \end{aligned} \tag{19}$$

$$\begin{aligned} A_1 &= \frac{k}{1!}\frac{d}{d\lambda}\left[\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^m\right]_{\lambda=0} \\ &= km\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^{m-1} \\ &\quad *\left(q_1 + 2q_2\lambda + 3q_3\lambda^2 + \cdots\right) \\ &= kmq_0{}^{m-1}q_1 = kmf^{m-1}q_1 \end{aligned} \tag{20}$$

$$A_2 = \frac{k}{2!}\frac{d^2}{d\lambda^2}\left[\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^m\right]_{\lambda=0}$$

$$= \frac{k}{2}\frac{d}{d\lambda}\begin{bmatrix} m\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^{m-1} * \\ \left(q_1 + 2q_2\lambda + 3q_3\lambda^2 + \cdots\right) \end{bmatrix}$$

$$= \frac{k}{2}\begin{bmatrix} m\left(m-1\right)\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^{m-2} * \\ \left(q_1 + 2q_2\lambda + 3q_3\lambda^2 + \cdots\right)^2 + \\ m\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^{m-1} * \\ \left(2q_2 + 6q_3\lambda + \cdots\right) \end{bmatrix}$$

$$= \frac{km}{2}\left[\left(m-1\right)\left(q_0\right)^{m-2}\left(q_1\right)^2 + \left(q_0\right)^{m-1}\left(2q_2\right)\right] \tag{21}$$

Close to what has been done, from a recurrence relation, one gets

$$q_{j+1} = q_0 + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^2}\right) + D_t^{-\alpha} A_j$$

$$q_0 = f$$

$$\begin{aligned} q_1 &= q_0 + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_0}{\partial x^2}\right) + D_t^{-\alpha} A_0 \\ &= f + d_f D_t^{-\alpha}\left(f''\right) + D_t^{-\alpha}\left(kf^m\right) \\ &= f + d_f f'' D_t^{-\alpha}(1) + kf^m D_t^{-\alpha}(1) \\ &= f + \left(d_f f'' + kf^m\right) D_t^{-\alpha}(1) \\ &= f + \left(d_f f'' + kf^m\right)\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \end{aligned} \tag{22}$$

$$\begin{aligned} q_1 &= q_0 + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_0}{\partial x^2}\right) + D_t^{-\alpha} A_0 \\ &= f + d_f D_t^{-\alpha}\left(f''\right) + D_t^{-\alpha}\left(kf^m\right) \\ &= f + d_f f'' D_t^{-\alpha}(1) + kf^m D_t^{-\alpha}(1) \\ &= f + \left(d_f f'' + kf^m\right) D_t^{-\alpha}(1) \\ &= f + \left(d_f f'' + kf^m\right)\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \end{aligned} \tag{23}$$

$$\begin{aligned} q_2 &= q_1 + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_1}{\partial x^2}\right) + D_t^{-\alpha} A_1 \\ &= f + \left(d_f f'' + kf^m\right)\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + d_f D_t^{-\alpha}\left(f'' + \left(d_f f'' + kf^m\right)''\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\right) \\ &+ D_t^{-\alpha}\left(kmf^{m-1}\left(f + \left(d_f f'' + kf^m\right)\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\right)\right) \\ &= f + \left(d_f f'' + kf^m\right)\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + d_f\left(f''\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + \left(d_f f'' + kf^m\right)''\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right) \\ &+ \left(kmf^{m-1}\left(f\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + \left(d_f f'' + kf^m\right)\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right)\right) \end{aligned} \tag{24}$$

Then

$$q = q_0 + q_1 + q_2$$

$$\begin{aligned} q &= 3f + 2\left(d_f f'' + kf^m\right)\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \\ &+ d_f\begin{pmatrix} f''\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \\ + \left(d_f f'' + kf^m\right)''\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right) \end{pmatrix} \\ &+ \left(kmf^{m-1}\begin{pmatrix} f\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \\ + \left(d_f f'' + kf^m\right)\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right) \end{pmatrix}\right) \end{aligned} \tag{25}$$

Discussion of results for Autocatalytic Reaction model

Concentration of the species function is deduced analytically by the method of ADM into the reaction function of power to “m” as obtained by equation (25). In these calculations, the physical parameters $d_f = 10^{-5}$ and $f = e^{-x}$ as an initial concentration function.

Visualizations shows the variation of concentration of the species function against to distance x for some values of fractional parameter” α ” as shown in Fig. (1) when $m = 2$, $k = 0.004$ and $t = 100.1$. On contrary when $m = 2$, $k = 0.005$, and $t = 100.1$ as shown in Fig. (2). It is noted that when α increases then the concentration decreases. The variation of concentration of the species function against to distance x for variant values of time “t” is shown in Fig. (3). It is noted that when time increases then the concentration is proportional inversely with the time values “t”. The variation of concentration of the species function against to distance x for variant values of diffusion parameter “ d_f ” is shown in Fig. (4). It is noted that the concentration proportional directly with diffusion parameter “ d_f ”. The variation of concentration of the species function against to distance x for different values of power parameter **“m”**, and diffusion coefficient “K” is shown in Figs. (5-6). It is noted that the concentration proportional inversely with both parameters.

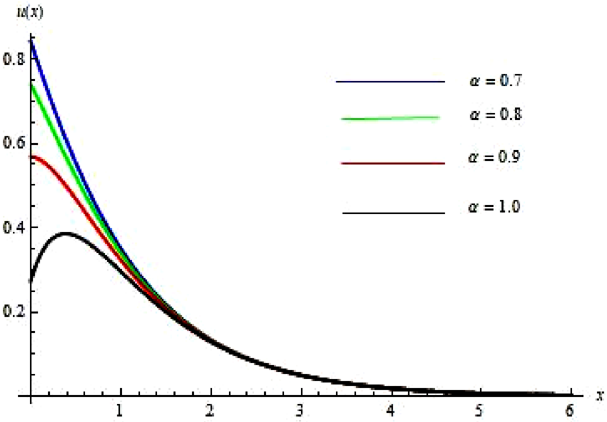

Fig. (1): The concentration varies against distance for some values of fractional parameter “ α ” when $m=2$, $d_f=10^{-5}$, $k=0.004$, and $t=100.1$.

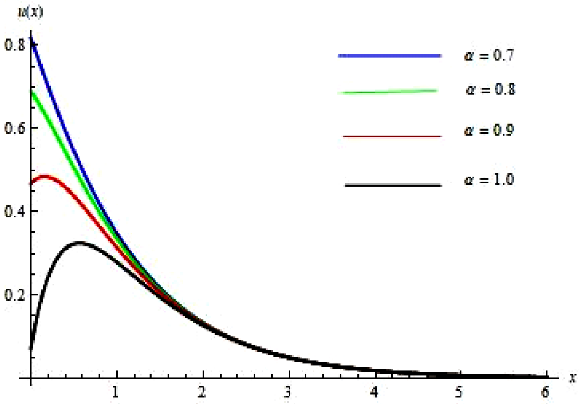

Fig. (2): The concentration varies against distance for some values of fractional parameter “ α ” when $m=2$, $d_f=10^{-5}, k=0.005$, and $t=100.1$.

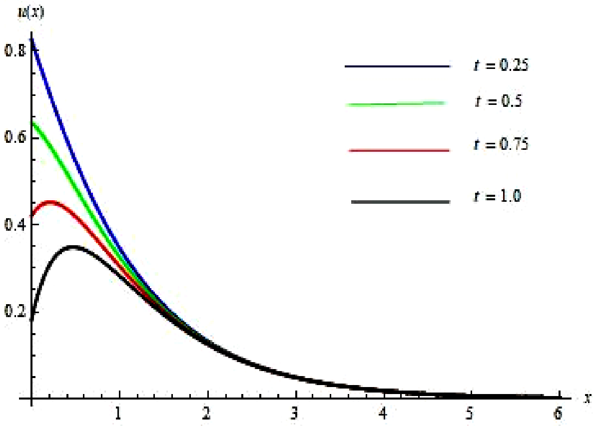

Fig. (3): The concentration varies with distance for some values of time t when $m=2$, $d_f=10^{-5}, k=0.5$, and $\alpha = 0.9$.

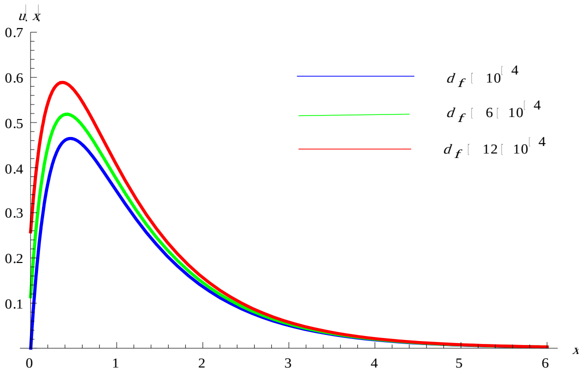

Fig (4): The concentration varies against distance for some values of diffusion parameter “ d_f ” when $t=100.1$, k=0.005, m=2, and $\alpha=0.9$.

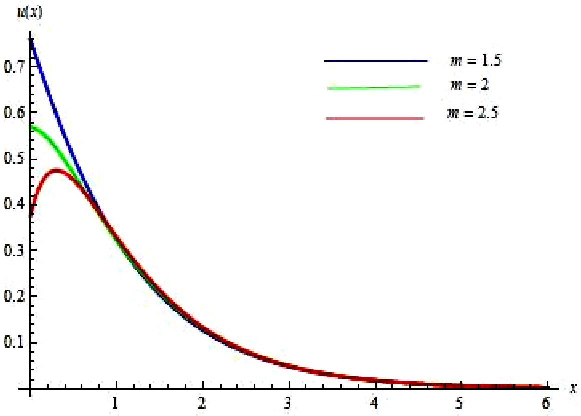

Fig. (5): The concentration varies against distance for some values of power parameter "m when $t = 100.1$, $k = 0.001$ and $\alpha = 0.9$.

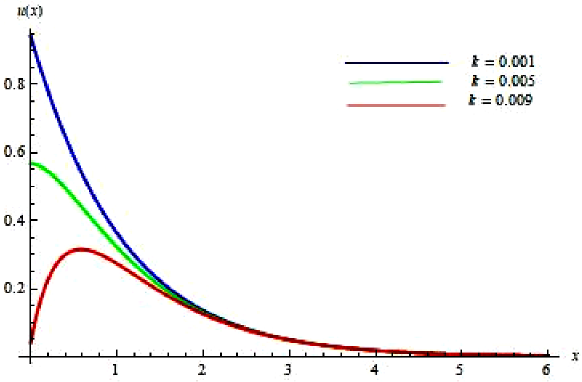

Fig. (6): The concentration varies against distance for some values of diffusion coefficient "K" when $t = 100.1, d_f = 10^{-5}$, , m=2 and $\alpha = 0.9$.

Adomian decomposition method for Bistable or Schlogl reaction model

The recurrence relation is given by:

$$q_0 = f \qquad (26)$$

$$q_{j+1} = D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^2}\right) + D_t^{-\alpha} A_j \qquad (27)$$

Introducing a parameter λ in the Adomian polynomials, then, in view of equations (22) and (23), we have:

The Adomian polynomials in case of Bistable model has the form:

$$q_1 - q_0 = D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^2}\right) + D_t^{-\alpha} A_j \qquad (28)$$

$$A_i = \frac{k}{\lambda_j}\left[\sum_0^\infty f_m q_j \lambda^j\right]_{\lambda=0} \qquad (29)$$

By applying the ADM we can write

$$q_{j+1} = D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^2}\right) + D_t^{-\alpha} A_j \qquad (30)$$

So that

$$A_i = \frac{1}{j!}\frac{d^j}{d\lambda^j}\left[\begin{array}{l} k_1 - k_2\left(\sum_0^\infty q_j \lambda^j\right) \\ +k_3\left(\sum_0^\infty q_j \lambda^j\right)^2 - k_4\left(\sum_0^\infty q_j \lambda^j\right)^3 \end{array}\right]_{\lambda=0}$$

$$R(q) = k_1 - k_2 q + k_3 q^2 - k_4 q^3 \qquad (31)$$

$$A_0 = \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[\begin{array}{l} k_1 - k_2\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right) \\ +k_3\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^2 \\ -k_4\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^3 \end{array}\right]_{\lambda=0}$$

$$A_0 = k_1 - k_2 q_0 + k_3 {q_0}^2 - k_4 {q_0}^3 \qquad (32)$$

$$A_1 = \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[\begin{array}{l} k_1 - k_2\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right) \\ +k_3\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^2 \\ -k_4\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^3 \end{array}\right]_{\lambda=0}$$

$$= \left[\begin{array}{l} -k_2\left(q_1 + 2q_2\lambda + \cdots\right) \\ +2k_3\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)\left(q_1 + 2q_2\lambda + \cdots\right) \\ -3k_4\left(q_0 + q_1\lambda + q_2\lambda^2 + \cdots\right)^2\left(q_1 + 2q_2\lambda + \cdots\right) \end{array}\right]_{\lambda=0}$$

$$A_1 = -k_2\left(q_1\right) + 2k_3\left(q_0\right)\left(q_1\right) - 3k_4\left(q_0\right)^2\left(q_1\right) \qquad (33)$$

Close to what has been done, from a recurrence relation, one gets

$$q_{j+1} = D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_j}{\partial x_i^{\,2}}\right) + D_t^{-\alpha} A_j \tag{34}$$

$$q_0 = f \tag{35}$$

$$\begin{aligned}
q_1 &= q_0 + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_0}{\partial x^2}\right) + D_t^{-\alpha} A_0 \\
&= f + d_f D_t^{-\alpha}\left(f''\right) \\
&\quad + D_t^{-\alpha}\left(k_1 - k_2 q_0 + k_3 {q_0}^2 - k_4 {q_0}^3\right) \\
&= f + \begin{pmatrix} d_f f'' + k_1 - k_2 q_0 \\ + k_3 {q_0}^2 - k_4 {q_0}^3 \end{pmatrix} D_t^{-\alpha}(1) \\
q_1 &= f + \begin{pmatrix} d_f f'' + k_1 - k_2 q_0 \\ + k_3 {q_0}^2 - k_4 {q_0}^3 \end{pmatrix}\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right)
\end{aligned} \tag{36}$$

$$\begin{aligned}
q_2 &= q_1 + D_t^{-\alpha}\left(\sum_{i=1}^{n} d_f \frac{\partial^2 q_1}{\partial x^2}\right) + D_t^{-\alpha} A_1 \\
&= f + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \\
&\quad + d_f D_t^{-\alpha}\left(f'' + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}'' \left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\right) \\
&\quad + D_t^{-\alpha}\left(-k_2 + 2k_3(f) - 3k_4(f)^2\right) \\
&\quad * \left(f + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\right)
\end{aligned}$$

$$\begin{aligned}
q_2 &= f + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \\
&\quad + d_f \left(f''\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}'' \left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right) \\
&\quad + \left(-k_2 + 2k_3(f) - 3k_4(f)^2\right) \\
&\quad * \left(f\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right)
\end{aligned} \tag{37}$$

Then

$$q = q_0 + q_1 + q_2$$

$$\begin{aligned}
q &= 3f + 2\begin{pmatrix} d_f f'' + k_1 - k_2 q_0 \\ + k_3 {q_0}^2 - k_4 {q_0}^3 \end{pmatrix}\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) \\
&\quad + d_f \left(f''\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}'' \left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right) \\
&\quad + \left(-k_2 + 2k_3(f) - 3k_4(f)^2\right) \\
&\quad * \left(f\left(\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + \begin{pmatrix} d_f f'' + k_1 - k_2 f \\ + k_3 f^2 - k_4 f^3 \end{pmatrix}\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right)
\end{aligned} \tag{38}$$

Discussion of results for Bistable or Schlogl reaction model

Concentration of the species function is obtained by using Adomain decomposition method for reaction function rating $R(q) = k_1 - k_2 q + k_3 q^2 - k_4 q^3$ in case of Bistable or Schlogl reactions by equation (35). In these calculations, the physical parameters $d_f = 10^{-5}$ and $f = e^{-x}$ as an initial concentration function.

The variation of concentration of the species function against to distance x is plotted for some fractional parameter values “α “as shown in Figs. (7-8). It is noted that when α increases then the concentration increases. The variation of concentration of the species functions against to distance x is plotted for different values of time t as shown in Fig. (9). It is noted that when time increases then the concentration increases. The variation of concentration of the species function against to distance x is plotted for different values of diffusion coefficient “d_f” is shown in Fig. (10). It is noted that concentration is proportional directly with diffusion coefficient.

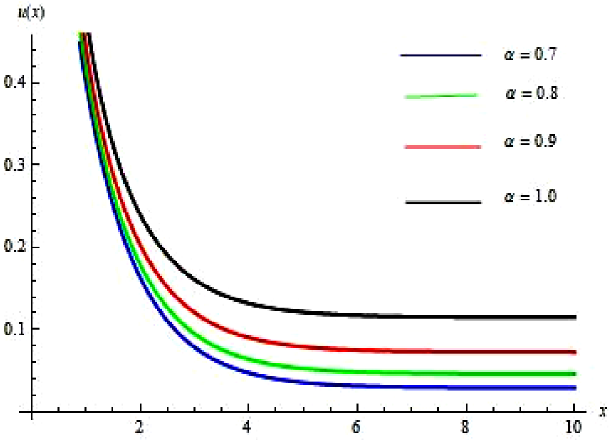

Fig. (7): The concentration varies against distance for some values of fractional order α when t=100.

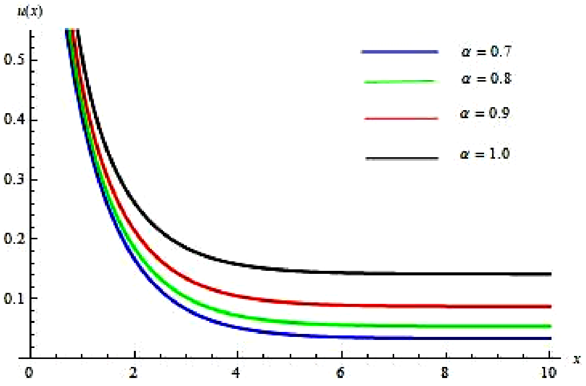

Fig. (8): The concentration varies against distance for some values of fractional order α when t=120.

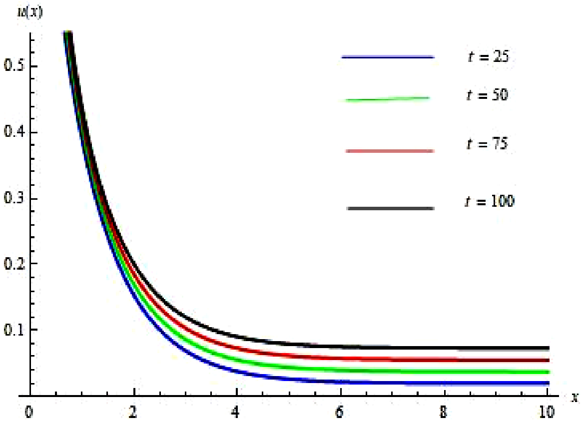

Fig. (9): The concentration varies against distance for some values of time when $\alpha = 0.9$.

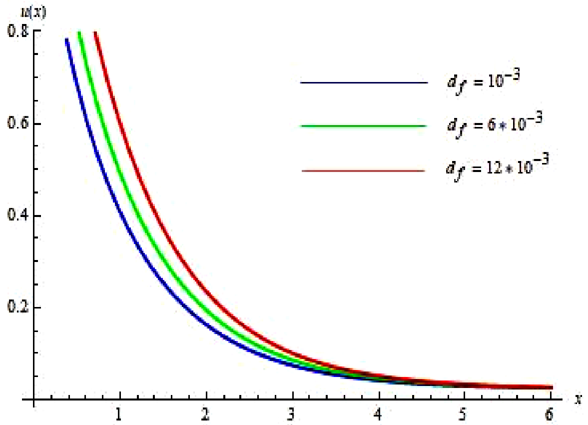

Fig. (10): The concentration varies against distance for some values of diffusion parameter “ d_f ”, $10^{-8} \le d_f \le 10^{-3}$ when $\alpha = 0.9$.

Conclusion

The conclusions points of the above results and plotted figures are as follows:

1. Concentration of the species function is regularly change with the variation of dominants parameters as fractional α time t, diffusion coefficient “ d_f ” power parameter **“m”**, and parameter **“K”**.
2. Concentration of the species function proportional inversely with fractional parameter, time, power factor “m”, coefficient **“K”** and directly with coefficient of diffusion “d_f” in case of Autocatalytic model.
3. The concentration of the species function is proportional directly with fractional parameter, time and with diffusion coefficient in case of “Bistable or Schlogl Model”.
4. The concentration is proportional directly with diffusion coefficient “d_f” in both cases.
5. The concentration is affected by the reaction term $R(u)$ which is considered as a dominate factor in the Autocatalytic

Reaction and Bistable or Schlogl Reaction Models.

6. The concluded results prove the validity of the proposed model.

7. The current work can be extended for other physical chemical reaction forms $R(u)$ as a future prospect.

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طريقة التحليل الأدومى لمعادلة الإنتشار والتفاعل الكسرى الزمنى

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في هذا البحث، نستكشف طريقة التحلل الأدومي (ADM)، وخاصةً قدرتها الرمزية على تحديد حلول المعادلات التفاضلية الجزئية. تُستخدم طريقة التحلل الأدومي لحل معادلة الانتشار والتفاعل الكسري الزمني. تُطبّق هذه الطريقة على نموذجين من معادلات الانتشار والتفاعل الكسري الزمني (التفاعل الذاتي التحفيزي Bistable، والنموذج ثنائي الاستقرار، ونموذج Schlogl). تُظهر نتائج هذه الطريقة أن فعالية الحل قريبة من المقترحات العملية لبحثنا، وهي قريبة من الحل الدقيق. علاوة على ذلك، تُقدّم الطريقة الحالية لحل معادلة الانتشار والتفاعل الكسري الزمني. يتناسب تركيز دالة الأنواع طرديًا مع جميع المعاملات الفيزيائية في "نموذج ثنائي الاستقرار أو شلوغل"، وعكسيًا في حالة "النموذج الذاتي التحفيزي" على التوالي. وقد تم إفتراض بأن يكون الحدان الأول والثاني من التحليل كافيين للحل التقريبي.