DJS Vol. 50(1) (2025) – pp. 1-10 ISSN: 1012-5965

Delta Journal of Science
Available online at
https://djs.journals.ekb.eg/

Research Article **MATHEMATICS**

Validation of Class Membership Degree in Information Systems using Similarity Rough Theory

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Received: 16/12/2024 **Accepted:** 4/2/2025

KEY WORDS

Membership function, degree of similarity, information system, uncertain idea, and similarity connection.

ABSTRACT

Accurately quantifying the relationships between conditional and decision attributes is crucial for information systems to make well-informed decisions. This work presents a novel approach to verifying class membership using Similarity Rough Set Theory (SRS) and a unique symmetry-based point of view. We apply SRS to model attribute interactions and their impact on overall class membership in detail. For determining the membership degree, two methods are provided: one that depends on individual attribute and the second method takes all attributes into consideration. The efficiency of the proposed work with respect to the existing one is clarified by presenting a case study of higher education university information system. The results illustrates that the membership degrees obtained by individual and aggregated attribute are effective methods, indicating a good relations of class membership. In addition, it shows the practical effectiveness of the proposed technique by applying it on a large dataset of academic student marks. This work contributes in evaluating and verifying the class membership for more informed decision in the real-life applications.

Introduction

Rough Set Theory (RST) is widely applied in handling data for various real-life applications, including rule extraction, knowledge discovery, and decision-making **(World Health Organization, 2022)**. Recently, RST has seen significant advancements, along with a growing need to analyze and process large and complex datasets **(Pawlak, 1991; Yao, 2018)**. It is regarded as a powerful tool for managing uncertainty in data, particularly within information systems.

However, traditional applications of RST are constrained by their reliance on equivalence relations. To address these limitations, numerous extensions of RST have been developed, utilizing binary or alternative types of relations. In 1998, Yao introduced methods that led to the development of various interpretations of rough sets based on different relations, including reflexive **(Abo-Tabl and El-Bably, 2022)**. Similarity **(Abo-Tabl, 2013; Dai et al., 2018)** tolerance **(Skowron and Stepaniuk, 1996)**, general relations **(Abu-Gdairi et al., 2021; El-Gayar and Abu-Gdairi, 2024)** and topological approaches **(El-Bably et al., 2024; El-Gayar et al., 2023; El-Gayar and El Atik, 2022; Hosny et al., 2024)**. RST applications have been explored in a variety of fields **(Abu-Gdairi and El-Bably et al., 2024; El-Bably et al., 2023; Abu-Gdairi et al., 2023; Taher et al., 2024a; Taher et al., 2024b)**. Similarity Rough Set Theory (SRS), an extension of RST, integrates similarity measurements into information frameworks, enabling a more flexible approach to modeling relationships between classes and data points **(Liu et al., 2022; Yao, 2003)**. SRS offers a more specialized and effective method for analyzing large datasets, where traditional RST techniques may fall short **(Hu et al., 2020; Zhang and Miao, 2004)**. Some key advantages of SRS in data-driven environments include:

- **Membership Determination**: SRS provides an efficient method for estimating membership degrees for classes, thereby enhancing decision-making capabilities **(Yao, 2021)**.
- **Enhanced Data Analysis**: SRS enables comprehensive analysis by capturing intricate relationships within complex and large data structures in information systems **(Zhang, 2020)**.
- **Performance**: SRS evaluates individual contributions of class elements with nuanced assessments, resulting in improved performance and accuracy **(Wu et al., 2023)**.

This paper contributes to information system analysis and decision-making through the following methods:

1. Developing a framework that offers a more accurate representation of attribute relationships compared to existing techniques.
2. Applying two membership calculation strategies:
   - Focus on individual attribute symmetry.
   - Consider the collective symmetry of all attributes.

This provides users with a flexible approach to select the best method based on the data's characteristics.

3. Conducting a comprehensive case study to demonstrate the framework's effectiveness. Specifically, we showcase how the proposed framework outperforms existing methods in identifying student performance levels within a university development information system, highlighting its practical utility and robustness.

The broader implications of this research extend beyond student performance analysis. Our framework is a versatile tool for evaluating and improving information systems across various domains, including healthcare, finance, manufacturing, and e-commerce. By equipping decision-makers with a clearer understanding of attribute relationships, this research facilitates more informed choices, enhanced system performance, and better outcomes across diverse applications.

The remainder of this paper is organized as follows:

- Section II discusses relevant literature and preliminaries.
- Section III introduces a novel dissimilarity-based approach for validating class memberships in information systems.
- Section IV presents a case study analyzing student grade data to evaluate the effectiveness of the proposed approach.

Section V concludes the paper and outlines future work.

PRELIMINARIES

Definition II.1 (Yao, 1998) Let U be the universe set and an equivalence relation S ⊆ U × U. Let A be a subset of U. Let S (x) be the equivalence class on U. The lower and upper approximations of A are given as:

$$S^{\downarrow}(A) = \bigcup_{x \in A} S(x) : S(x) \subseteq A$$

$$S^{\uparrow}(A) = \bigcup_{x \in A} S(x) : S(x) \cap A = \emptyset$$

$$S^{\downarrow}(A) \subseteq A \subseteq S^{\uparrow}(A)$$

Boundary region of A

$$BN_S(A) = S^{\uparrow}(A) - S^{\downarrow}(A)$$

$$BN_S(A) = \{S13\}$$

The pair $S^{\downarrow}(A), S^{\uparrow}(A)$ is called a rough set.

<u>Example II.1:</u>

Table (1) contains IS data about 20 students, including their grades in coursework, midterm exams, final exams, and their final overall grade.

$U = \{S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15, S16, S17, S18, S19, S20\}$

$U/S = \{\{S1\}, \{S2\}, \{S3\}, \{S4\}, \{S5, S13\}, \{S6\}, \{S7\}, \{S8\}, \{S9\}, \{S10\}, \{S11, S15\}, \{S12\}, \{S16\}, \{S17\}, \{S18, S19\}, \{S20\}\}$

$X = \{S2, S3, S4, S5\}$

$S^{\downarrow}(A) = \{S2, S3, S4\}$,

$S^{\uparrow}(A) = \{S2, S3, S4, S5, S13\}$

Boundary region of A:

$BN_S(A) = S^{\uparrow}(A) - S^{\downarrow}(A) = \{S5, S13\}$

The pair $S^{\downarrow}(A), S^{\uparrow}(A)$ is called a rough set. Figure (1) illustrates the rough set and boundaries.

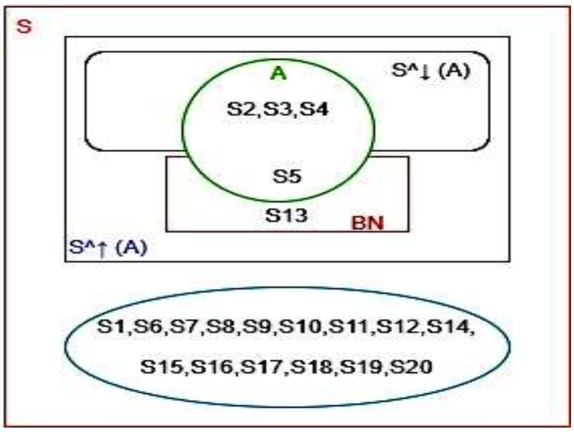

Fig. (1): Rough set and Boundaries

Table (1): IS data about 20 students

Student ID	Coursework	Midterm	Final Exam	Final Grade
S1	D	F	C	D
S2	A	A	A	A
S3	A	B	A	A
S4	C	A	A	A
S5	C	A	C	A
S6	D	C	C	C
S7	A	C	A	B
S8	D	A	B	C

Definition II.2 (Skowron and Rauszer, 1992): Let an information system, denoted as IS = (U, A), is a tuple consisting of:

U: A non-empty finite set of objects called the universe. This represents the entities or data points within the system.

$U = \{S1, S2, S3, S4, \ldots\ldots, Sn\}$

A: A non-empty finite set of attributes called the attribute set. These are the features or characteristics used to describe the objects in the universe.

$A = \{a1, a2, a3, a4, \ldots\ldots, an\}$

V: The set *Va* is called the value set of *a* such that U → *Va*, *a* ϵ *A*. The set of attributes can be classified in two subsets *C* ⊆ *A* and *D* = *A* – *C* where:

C: A subset of A called the conditional attribute set. These are the attributes used to predict or categorize the objects in the universe.

D: The complement of C in A, also called the decision attribute set. This is the attribute or set of attributes that represent the target variable or class labels of the objects.

Example II.2:

For our IS data that illustrates in Table (1),

$U = \{S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15, S16, S17, S18, S19, S20\}$

$A = \{\text{Coursework, Midterm, Final Exam, Final Grade}\}$

$C = \{\text{Coursework, Midterm, Final Exam}\}$

$D = \{\text{Final Grade}\}$

Definition II.3. (Pawlak, 1998) Assume IS = (U, A) is an information system and ϕ ≠ S ⊆ U. The Rough membership function for the set X is

$$\mu_S^X(s) = \frac{|\,[s]_X \cap S\,|}{[s]_X} \quad \text{for some } s \in U$$

Example II.3. Let IS = (U, X) be an information system that shown in Table (1). Let

$U = \{S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15, S16, S17, S18, S19, S20\}$

$X = \{S2, S3, S4, S5\}$ Let the students who has final grade “A”

$[s]_X = \{$ C, A, C} this is the chosen conditional attributes such that coursework “C”, Midterm “A”, and Final Exam “C”. Repeat for $[s]_X = \{$F, F, F}

$$\mu_S^X(S5) = \frac{|\,\{S5, S13\} \cap \{S2, S3, S4, S5\}\,|}{\{S5, S13\}} = \frac{1}{2}$$

For $[s]_A = \{$ F, F, F}, it is the chosen conditional attributes such that coursework “F”, Midterm “F”, and Final Exam “F”.

$$\mu_S^X(S18) = \frac{|\,\{S18, S19\} \cap \{S2, S3, S4, S5\}\,|}{\{S18, S19\}} = 0$$

Definition II.4 (Rico et al., 2022): To calculate the dissimilarity between the two things, let’s say that ak attributes for two elements (i,j) described by aki, akj. Matches for these two objects define the degree of dissimilarity between them:

$$\delta_{kij}(a_{ki}, a_{kj}) = \begin{cases} 1 & , if\ a_{ki} \neq a_{kj} \\ 0 & , if\ a_{ki} = a_{kj} \end{cases}$$

Example II.4. For S11 and S12 in Table (1), we can calculate the similarity between these two students as follows:

Student ID	Coursework	Midterm	Final Exam	Final Grade
S11	C	A	D	B
S12	C	B	D	C

Student ID	S11	S12
S11	3	2
S12	2	3

MEMBERSHIP BASED ON SIMILARITY

This objective of this paper is to generate a matrix that represents the dissimilarity relation using different attribute choice sets, and then use the dissimilarity degrees to generate a membership function for scenarios involving multi-class scenarios.

Definition III.1

Let us define

1- For given IS data U
$U = \{S1, S2, S3, S4, \ldots\ldots, SN\}$

2- The attribute set.
$A = \{a1, a2, a3, \ldots\ldots, aM\}$

3- The selected attribute decision set
$$A_S = \{a1, a2, a3, \ldots\ldots, aK\}$$
$$1 \le K \le M$$

4- The dis-similarity relation between two elements (i,j) for certain attribute (k).
$\delta_{kij}(a_{ki}, a_{kj})$

5- The total dis-similarity weight between the elements (i,j) for the selected attributes decision set
$$\omega(Si, Sj) = \sum_{k=1}^{K} \delta_{kij}(a_{ki}, a_{kj}) \quad , i \neq j$$

6- The information class set that the rough membership function will be calculated
$$X = \{S1, S3, \ldots\ldots, SN\}$$

Such that $\{S1, S3, \dots \dots, SN\}$ are common in certain decision attribute.

7- The membership function of the class X based on the total dis-similarity weight values for each element in information system

$$\mu_X = \frac{\sum_{S_i \in X \cap U(S)} \omega(S,Si)}{\sum_{S_i \in U(S)} \omega(S,Si)} \quad ,S \neq Si$$

Case study Evaluation

In this section we will apply and enable informed decision-making through accurate membership degree function calculation by using the novel dis-similarity-based method for validating element class membership in information systems.

Example IV.1

Regarding the data system depicted in Table 1, we have 20 student data set U:

$$U = \{S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15, S16, S17, S18, S19, S20\}$$

With set of attributes

$$A = \{\text{Coursework}, \text{Midterm}, \text{Final Exam}, \text{Final Grade}\}$$

Case (1) Individual Attribute Membership Degree Evaluation

For each individual attribute decision set, we calculate its dis-similarity matrix to obtain the membership function of certain class set. We firstly will apply the selected attribute decision set for first attribute such that: $A_S = \{\text{Coursework}\}$

Attribute Dis-similarity Matrix Calculation: We create a matrix that captures the dis-similarities between an attribute. This matrix quantifies how different attributes are from each other, providing a foundation for membership function evaluation. Table (2) shows coursework dis-similarity matrix.

Table (2): Coursework Dis-similarity Matrix

ID	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20
S1	0	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0	1	1	1	1
S2	1	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1	1	1
S3	1	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1	1	1
S4	1	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1
S5	1	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1
S6	0	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0	1	1	1	1
S7	1	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1	1	1
S8	0	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0	1	1	1	1
S9	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	1
S10	1	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1
S11	1	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1
S12	1	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1
S13	1	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1
S14	1	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1	1	1
S15	1	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1
S16	0	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0	1	1	1	1
S17	1	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1	1	1
S18	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	1
S19	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	1
S20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Using the dis-similarity matrix, we get:

$$U(s) = \{\omega : \delta(a_{ki}, a_{kj})\} \leq \lambda$$

We select classes in multiple ways based on the significance of λ

Secondly will apply the selected attribute decision set for second attribute such that:

$$A_S = \{\text{Midterm}\}$$

Table (3) shows the Midterm dis-similarity matrix for all elements in Table (1).

Third, we will apply the selected attribute decision set for third attribute such that:

$$A_S = \{\text{Final Exam}\}$$

Table (4) shows the final exam dis-similarity matrix for all elements.

Now based on Definition III.1 we calculate proposed membership degree and Pawlak's degree for each attribute dis-similarity matrix individually for $\lambda \leq 1$ and the selected information class set X is the students who has final grade "B" such that:

$$X = \{S7, S11, S13\}$$

Table (3): Midterm Dis-similarity Matrix

ID	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20
S1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
S2	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S3	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
S4	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S5	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S6	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
S7	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
S8	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S9	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S10	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1
S11	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S12	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
S13	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S14	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1
S15	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S16	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
S17	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
S18	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
S19	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
S20	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0

Table (5) shows the membership degree and Pawlak's degree for each attribute dis-similarity matrix individually and illustrates how our proposed method is accurate compared with Pawlak's degree that displays same value along all data elements.

Table (4): Final Exam Dis-similarity Matrix

ID	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20
S1	0	1	1	1	0	0	1	1	1	0	1	1	0	1	1	0	1	1	1	1
S2	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
S3	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
S4	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
S5	0	1	1	1	0	0	1	1	1	0	1	1	0	1	1	0	1	1	1	1
S6	0	1	1	1	0	0	1	1	1	0	1	1	0	1	1	0	1	1	1	1
S7	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
S8	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
S9	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	1	1	1	1	0
S10	0	1	1	1	0	0	1	1	1	0	1	1	0	1	1	0	1	1	1	1
S11	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	1	1	1	1	0
S12	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	1	1	1	1	0
S13	0	1	1	1	0	0	1	1	1	0	1	1	0	1	1	0	1	1	1	1
S14	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	1	1	1	1	0
S15	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	1	1	1	1	0
S16	0	1	1	1	0	0	1	1	1	0	1	1	0	1	1	0	1	1	1	1
S17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1
S18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1
S19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1
S20	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	1	1	1	1	0

Table (5): The Membership Degree and Pawlak's Degree for each attribute Dis-similarity Matrix individually

	Coursework		Midterm		Final Exam	
ID	Proposed	Pawlak	Proposed	Pawlak	Proposed	Pawlak
S1	0.19	0.15	0.2	0.15	0.14	0.15
S2	0.13	0.15	0.09	0.15	0.13	0.15
S3	0.13	0.15	0.17	0.15	0.13	0.15
S4	0.08	0.15	0.09	0.15	0.13	0.15
S5	0.08	0.15	0.09	0.15	0.14	0.15
S6	0.19	0.15	0.11	0.15	0.14	0.15
S7	0.13	0.15	0.11	0.15	0.13	0.15
S8	0.19	0.15	0.09	0.15	0.16	0.15
S9	0.18	0.15	0.09	0.15	0.14	0.15
S10	0.08	0.15	0.17	0.15	0.14	0.15
S11	0.08	0.15	0.09	0.15	0.14	0.15
S12	0.08	0.15	0.17	0.15	0.14	0.15
S13	0.08	0.15	0.09	0.15	0.14	0.15
S14	0.13	0.15	0.17	0.15	0.14	0.15
S15	0.08	0.15	0.09	0.15	0.14	0.15
S16	0.19	0.15	0.09	0.15	0.14	0.15
S17	0.13	0.15	0.2	0.15	0.18	0.15
S18	0.18	0.15	0.2	0.15	0.18	0.15
S19	0.18	0.15	0.2	0.15	0.18	0.15
S20	0.16	0.15	0.2	0.15	0.14	0.15

Case (2) Aggregated Attributes Membership Degree Evaluation

We will apply the selected attribute decision set for all attribute such that:

$A_S = \{\text{Coursework, Midterm, Final Exam}\}$

Table (6) shows all attributes dis-similarity matrix for all elements in Table (1).

Based on Definition III.1 we calculate proposed membership degree and Pawlak's degree for all attribute dis-similarity matrix for $\lambda \leq 3$ and the selected information class set X is the students who has final grade "B" such that:

$$X = \{S7, S11, S13\}$$

Compared to earlier approaches, the idea presented in this paper offers a new way to find the function of membership by calculating the average of degrees of symmetry by either individual attribute or by all attributes. This approach results in more accurate approximation evaluation and decision-making.

$$\mu_{Average} = \frac{\mu_{CW} + \mu_{MT} + \mu_{FN}}{3}$$

Table (6): All attributes Dis-similarity Matrix

ID	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20
S1	0	3	3	3	2	1	3	2	3	2	3	3	2	3	3	1	2	2	2	2
S2	3	0	1	1	2	3	1	2	2	3	2	3	2	2	2	2	2	3	3	3
S3	3	1	0	2	3	3	1	3	3	3	3	2	3	2	3	3	2	3	3	3
S4	3	1	2	0	1	3	2	2	2	2	1	2	1	3	1	2	3	3	3	3
S5	2	2	3	1	0	2	3	2	2	1	1	2	0	3	1	1	3	3	3	3
S6	1	3	3	3	2	0	2	2	3	2	3	3	2	3	3	1	3	3	3	3
S7	3	1	1	2	3	2	0	3	3	3	3	3	3	2	3	3	2	3	3	3
S8	2	2	3	2	2	2	3	0	2	3	2	3	2	3	2	1	3	3	3	3
S9	3	2	3	2	2	3	3	2	0	3	1	2	2	2	1	2	3	2	2	2
S10	2	3	3	2	1	2	3	3	3	0	2	2	1	2	2	2	3	3	3	3
S11	3	2	3	1	1	3	3	2	1	2	0	1	1	2	0	2	3	3	3	2
S12	3	3	2	2	2	3	3	3	2	2	1	0	2	2	1	3	3	3	3	2
S13	2	2	3	1	0	2	3	2	2	1	1	2	0	3	1	1	3	3	3	3
S14	3	2	2	3	3	3	2	3	2	2	2	2	3	0	2	3	2	3	3	2
S15	3	2	3	1	1	3	3	2	1	2	0	1	1	2	0	2	3	3	3	2
S16	1	2	3	2	1	1	3	1	2	2	2	3	1	3	2	0	3	3	3	3
S17	2	2	2	3	3	3	2	3	3	3	3	3	3	2	3	3	0	1	1	2
S18	2	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	1	0	0	2
S19	2	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	1	0	0	2
S20	2	3	3	3	3	3	3	3	2	3	2	2	3	2	2	3	2	2	2	0

Table (7): The Membership Degree and Pawlak's Degree for all attribute Dis-similarity Matrix.

	All Attributes	
ID	Proposed	Pawlak
S1	0.18	0.15
S2	0.12	0.15
S3	0.14	0.15
S4	0.1	0.15
S5	0.11	0.15
S6	0.15	0.15
S7	0.12	0.15
S8	0.15	0.15
S9	0.14	0.15
S10	0.13	0.15
S11	0.11	0.15
S12	0.13	0.15
S13	0.11	0.15
S14	0.15	0.15
S15	0.11	0.15
S16	0.15	0.15
S17	0.17	0.15
S18	0.18	0.15
S19	0.18	0.15
S20	0.17	0.15

By comparison between calculated average membership degree for individual attributes in case 1 and calculated membership degree for all attributes case (2) we found that they are approximately equal as shown below in Table (8) and Fig. (2):

Table (8): Comparison between calculated Average/ All attributes Membership Degrees

ID	μ-all attributes	μ-CW	μ-MT	μ-FN	μ-Average
S1	0.18	0.19	0.2	0.14	0.18
S2	0.12	0.13	0.09	0.13	0.12
S3	0.14	0.13	0.17	0.13	0.14
S4	0.1	0.08	0.09	0.13	0.1
S5	0.11	0.08	0.09	0.14	0.1
S6	0.15	0.19	0.11	0.14	0.15
S7	0.12	0.13	0.11	0.13	0.12
S8	0.15	0.19	0.09	0.16	0.15
S9	0.14	0.18	0.09	0.14	0.14
S10	0.13	0.08	0.17	0.14	0.13
S11	0.11	0.08	0.09	0.14	0.1
S12	0.13	0.08	0.17	0.14	0.13
S13	0.11	0.08	0.09	0.14	0.1
S14	0.15	0.13	0.17	0.14	0.15
S15	0.11	0.08	0.09	0.14	0.1
S16	0.15	0.19	0.09	0.14	0.14
S17	0.17	0.13	0.2	0.18	0.17
S18	0.18	0.18	0.2	0.18	0.19
S19	0.18	0.18	0.2	0.18	0.19
S20	0.17	0.16	0.2	0.14	0.17

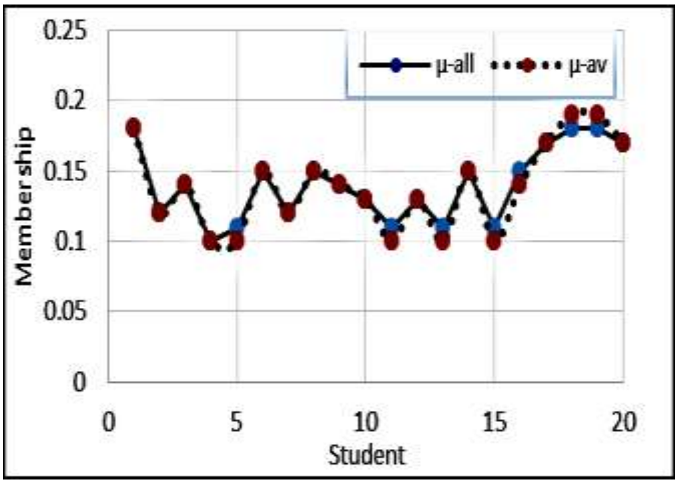

Fig. (2): Comparison between calculated Average/ All attributes Membership Degrees.

Furthermore, we showcase the practical utility of our method by successfully identifying Membership Degree within a large dataset of academic grades contains 200 students as shown in Fig. (3).

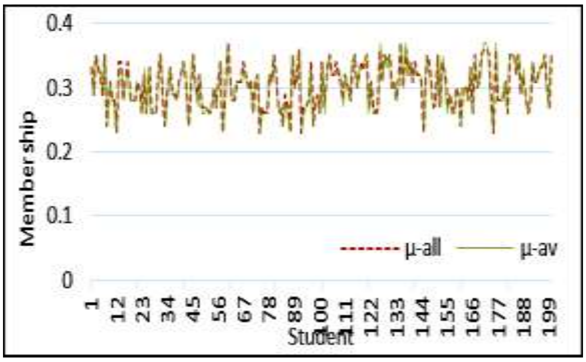

Fig. (3): Comparison between calculated Average/ All attributes Membership Degrees for large data set

CONCLUSION

This paper presents a novel SRS-based method for validating element class membership in information systems. Our approach demonstrates remarkable efficacy in both capturing attribute relationships and calculating membership degrees, even when compared to existing techniques. The similarity between individual and aggregate membership calculations adds robustness, while the case study and large-scale student performance evaluation highlight the practical feasibility and effectiveness of our method. This nuanced, context-aware approach has the potential to empower informed decision-making and performance improvement across various information systems, paving the way for future applications with enhanced accuracy and better-informed decisions. In our future work, we will discuss the proposed methods in other fields such as **(Abd El-Monsef et al., 2014; Abd El-Monsef et al., 2017; Al-Shami et al., 2020; Lu et al., 2021)**.

Acknowledgements

The authors wish to express their profound gratitude to the Editor-in-Chief, and the reviewers for their invaluable feedback and constructive suggestions, which played a critical role in enhancing the quality and clarity of this work. We would also like to extend our sincere thanks to our distinguished colleagues at the Egyptian School of Topology in Tanta, Egypt, founded by Prof. A. S. Mashhour and Prof. M. E. Abd El-Monsef, and currently led by Prof. A. M. Kozae. This institution has been a nurturing ground for numerous distinguished scholars both within Egypt and internationally, including those from Yemen, Palestine, Jordan, and Saudi Arabia. The authors are particularly appreciative of the school's steadfast support, encouragement, and the enriching intellectual exchanges that contributed significantly to this research. Finally, this work is dedicated to the memory of Prof. A. S. Mashhour, Prof. M. E. Abd El-Monsef, and Prof. A. Kandil— may their souls rest in peace.

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"التحقق من درجة انتماء الفئة في نظم المعلومات باستخدام نظرية التقريبات التشابهية"

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عند قياس العلاقات بدقة بين السمات الشرطية وسمات القرار أمرًا بالغ الأهمية لأنظمة المعلومات لاتخاذ قرارات مستنيرة. يقدم هذا العمل نهجًا جديدًا للتحقق من عضوية الفئة باستخدام نظرية المجموعات التقريبية للتشابه (SRS) ومنظور فريد يعتمد على التماثل. نقوم بتطبيق SRS لنمذجة التفاعلات بين السمات وتأثيرها على عضوية الفئة بشكل مفصل. لتحديد درجة العضوية، يتم تقديم طريقتين: الأولى تعتمد على السمات الفردية، والطريقة الثانية تأخذ جميع السمات بعين الاعتبار. يتم توضيح كفاءة العمل المقترح مقارنة بالنهج الحالي من خلال تقديم دراسة حالة لنظام معلومات جامعي للتعليم العالي. تُظهر النتائج أن درجات العضوية التي تم الحصول عليها باستخدام السمات الفردية والمجمعة هي طرق فعالة، مما يشير إلى علاقات جيدة لعضوية الفئة. بالإضافة إلى ذلك، تُبرز الفعالية العملية للتقنية المقترحة من خلال تطبيقها على مجموعة بيانات كبيرة لعلامات الطلاب الأكاديميين. يُسهم هذا العمل في تقييم والتحقق من عضوية الفئة لاتخاذ قرارات أكثر استنارة في التطبيقات الواقعية.