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Research Article

MATHEMATICS

Peristaltic flow of incompressible $Al_2o_{3/}$ water nanofluid under the effect of heat addition/absorption inside a vertical cylindrical tube

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KEY WORDS

Peristaltic vertical tube, $Al_2o_{3/}$water nanofluid, Newtonian fluid, Grashof number, Navier-Stokes equations, pressure gradient distributions.

ABSTRACT

The behavior of some peristaltic flow for some liquids in a vertical and horizontal cylindrical tube is studied before in non-closed form. In this paper, the $Al_2O_{3/}$ water nanofluid flow is discussed in a peristaltic vertical cylindrical tube under the effect of gravity and the constant heat addition/absorption. The mass, Navier-Stokes, heat and volume flow rate equations represent the mathematical model for long wavelengths. The system of equations represents a closed form for the dependent parameters. The model is solved analytically in terms of nano temperature, velocity and pressure gradient distributions with existence of nanoparticles suspended in water. The stream function is obtained in the plane of fluid with nanoparticles. The nano temperature distribution in a peristaltic tube is affected by different values of amplitude ratio and heating source parameter. The nano fluid velocity is increasing with the change of Grashof number values, rate of volume flow, heating source parameter and decreasing with void fraction of nano particles. The obtained results are in a closed form and prove the validity of the current model.

Introduction

Many scientists and researchers studied the transport of peristaltic flow through tubes, channels due to their important applications in engineering and medical sciences such as physiology, and finger pumps and roller, sanitary fluid transport, transport of corrosive fluids. Many researchers using the basic concepts and theories of fluid mechanics **(Landau and Lifshitz, 1987).** Moreover, the description of physical problems needs to suitable mathematical models. The fluid mechanics and heat mass exchange in two-phase flow is very important topic in industry and physical problems **(Bilicki et al., 1996)**. The two-phase flow can be studied for one and two components for superheated liquids and mixture of gas and liquid respectively. Moreover, the heat and mass exchange in the liquid-vapor represents the no equilibrium between two-phase-flow. The physiologists defined Peristalsis as a one of the major mechanisms for fluid transfer in many biological systems. The peristaltic motion in two-phase density flow and heat transfer affected the behavior of vapour bubbles inside a vertical cylindrical Tube **(Mohammadein et al.**, **2017**)**.**

Peristaltic flow of a Newtonian fluid in a porous medium surrounded vapor bubble in a curved channel l **(Mohammadein et al., 2019)**. Recently, in biophysics, the peristaltic flow for a mixture of blood and gas bubbles in the vertical inferior mesenteric vein is studied in human colon **(Elbendary and Mohammadein, 2022)**.This study called two-phase and two components. The effect of peristaltic motion on (CuO/Water) nanofluid inside a vertical cylindrical Tube **(Mohammadein et al., 2023)**.

In nanotechnology, a particle is defined as a small object that behaves as a whole unit with respect to its transport and properties. These particles are classified according to diameter. Nanoparticles used in nanofluid preparation usually have diameters below 100 nm. Moreover, the particles as small as 10 nm have been used in nanofluid research. When particles are not spherical but rod or tube-shaped, the diameter is still below 100 nm, but the length of the particles may be on the order of micrometers. It should also be noted that due to the clustering phenomenon, particles may form clusters with sizes on the order of micrometers. There are many nanoparticles used in industry like Al_2O_3, Cu, CuO, and Ag with different properties. The collations of one and two nanoparticles for a viscosity of concentrated suspensions and solutions are extended by **(Brinkman, 1952)**. New treatment of fluid mechanics with heat and mass transfer: The nonlinear Burger and Navier-Stokes are converted to a linear equations (**Mohammadein, 2020).**The simplest analytical solution of linear Navier-Stokes equations is introduced in two dimensions **(Mohammadein et al., 2021)**. The analytical and simplest resolution of linear Navier-Stokes equations in two dimensions **(Mohammadein et al., 2022)**.

Moreover,the obtained solution satisfied the original linear and nonlinear Navier-stokes equations.

The density and specific heat of nanofluid can be calculated by the following relations:

$$(\rho)_{nf} = \varphi_p (\rho)_p + (1 - \varphi_p)(\rho)_l \qquad (1)$$

$$(\rho c_p)_{nf} = \varphi_p (\rho c_p)_p + (1 - \varphi_p)(\rho c_p)_l \qquad (2)$$

On contrary the viscosity of particle suspensions was extended by **(Brinkman, 1952)**

$$(\mu)_{nf} = \mu_f (1 - \varphi_p)^{-2.5} \qquad (3)$$

Moreover, the effective thermal conductivity is given by

$$k_{eff} = \frac{(k_p + 2k_f) - 2\varphi_p (k_f - k_p)}{(k_p + 2k_f) + \varphi_p (k_f - k_p)} k_f \qquad (4)$$

In this paper, the flow of incompressible Newtonian $Al_2o_{3/}$ water nanofluid in a peristaltic vertical cylindrical tube is studied. The present model is represented by mass, Navier-Stokes, heat and volume flow rate equations for long wavelengths. The nano temperature, velocity and pressure gradient are obtained analytically and described by graphs with the effect of some dominant variables.

Analysis

A current problem introduces the motion of an incompressible viscous Newtonian Al_2o_3/ water nanofluid in a peristaltic vertical cylindrical tube. The flow represents a sinusoidal wave train propagating with constant speed along the boundaries of the outer tube.

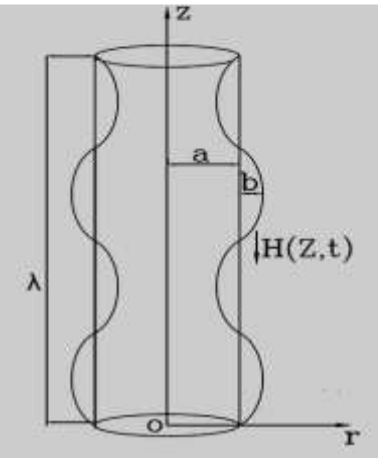

Fig. (1): Sketch of model

The axisymmetric cylindrical polar coordinate system is chosen such that the coordinate is along the center line of the tube in z-direction and coordinate along the radial coordinate in r-direction. The boundary of the tube is adjusted at a finite temperature and at the center we have used axisymmetric condition on temperature as proposed in the current problem. The physical model of the present problem is shown in Fig. 1, where "a" is the radius of the tube, "b" is the amplitude of the wave, λ is the wavelength and t is the time. The fluid flow is unsteady in the fixed frame

$(\bar{r}, \bar{z})$.

However, in a coordinate system moving with the propagation velocity c.

$$H = a + b\sin(\frac{2\pi}{\lambda}\bar{z}) \qquad (5)$$

The differential equations of mass, Navier-Stokes, and heat transfer represent the mathematical model of the current problem under the effect of constant heat addition/absorption in frame $(\bar{r}, \bar{z})$ as follows:

Mass Equation:

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\,\bar{u}) + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \qquad (6)$$

Navier- Stokes Equations in directions r and z respectively in the form:

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}\right) = -\frac{\partial \bar{p}}{\partial \bar{r}} + \mu_{nf}\left\{\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{u}}{\partial \bar{r}}\right) - \frac{\bar{u}}{\bar{r}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right\} \qquad (7)$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}\right) = -\frac{\partial \bar{p}}{\partial \bar{z}} + \mu_{nf}\left\{\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{w}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right\} + \rho_{nf} g\alpha(T - T_0) \qquad (8)$$

Heat Equation:

$$\rho_{nf}(c_p)_{nf}\left(\bar{u}\frac{\partial T}{\partial \bar{r}} + \bar{w}\frac{\partial T}{\partial \bar{z}}\right) = k_{nf}\left(\frac{\partial^2 T}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial T}{\partial \bar{r}} + \frac{\partial^2 T}{\partial \bar{z}^2}\right) + Q_0 \qquad (9)$$

Where" P" is the pressure of fluid, "T" is the fluid temperature, Q_0 is the constant heat absorption/addition, $(c_p)_{nf}$ is the nano specific heat at constant pressure, k is the thermal

$$\bar{r} = ar,\ \bar{z} = \lambda z,\ \bar{w} = cw,\ \bar{u} = c\delta u,\ \bar{p} = \frac{\mu c\lambda}{a^2}p,\ \bar{t} = \frac{\lambda}{c}t$$

$$\bar{\theta} = \frac{T - T_0}{T_0},\quad \bar{\delta} = \frac{a}{\lambda},\quad h = 1 + e\sin(2\pi z),\quad e = \frac{b}{a},$$

$$G_c = \frac{a^3 g\alpha T_0}{\nu^2},\qquad R_e = \frac{\rho_{nf} ca}{\mu},$$

$$Pr = \frac{\mu(c_p)_{nf}}{k}\quad \beta = \frac{a^2 Q_0}{kT_0}, \qquad (10)$$

where, R_e is the Reynolds number, e is the amplitude ratio, G_c is the Grashof number, Pr is the Prandtl number, and β_{nf} is the non-dimensional heat source parameter. Substituting from equation (10) into the equations (6-9), we obtain the following system in frame (r, z)

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u) + \frac{\partial w}{\partial z}$$

$$R_e\delta^3\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial r} + \delta^2\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2} + \delta^2\frac{\partial^2 u}{\partial z^2}\right\} \qquad (12)$$

$$\frac{\rho_{nf} c^2 a}{\mu c}\delta\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left\{\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \delta^2\frac{\partial^2 w}{\partial z^2}\right\} + \rho_{nf} g\alpha T_0\theta \qquad (13)$$

$$\rho_{nf}(c_p)_{nf} T_0 c\left(\delta u\frac{\partial \theta}{\partial r} + \delta w\frac{\partial \theta}{\partial z}\right) = \frac{(K_l)_{nf} T_0}{a^2}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \delta^2\frac{\partial^2 \theta}{\partial z^2}\right) + Q_0 \qquad (14)$$

The above system (11-14) is solved for large values of wavelength ($\delta \ll 1$) in the mixture of Al_2O_3/ water nanofluid. Then the mass equation still same but in contrary the equations (12-14) become

$$\frac{\partial P}{\partial r} = 0 \qquad (15)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = \mu_1\frac{\partial P}{\partial z} - G\theta_{nf} \qquad (16)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_{nf}}{\partial r}\right) = -\beta_{nf}. \qquad (17)$$

where, $\beta_{nf} = \beta \frac{k_f}{k_{nf}}$ and $G = \frac{\rho_{nf}}{\rho_l R_e} \mu_1 G_c$, $\mu_1 = \frac{\mu_l}{\mu_{nf}}$

The corresponding dimensionless boundary conditions for solving of equation (13) has the form

$$\frac{\partial \theta_{nf}}{\partial r} = 0 \quad at \quad r = 0 ,$$

$$\theta_{nf} = \theta_0 \quad at \quad r = h. \qquad (18)$$

then the nano temperature becomes

$$\theta_{nf}(r,z) = \theta_0 + \frac{\beta_{nf}}{4}(h^2 - r^2) \qquad (19)$$

Substituting from Eqn. (19) into the Eqn. (16) and solving equation (16) with the following boundary conditions

$$\frac{\partial w}{\partial r} = 0 \quad at \quad r = 0$$

$$w = A_0 \quad at \quad r = h \qquad (20)$$

then, the nano velocity components of nano fluid through a peristaltic tube becomes

$$W(r,z) = A_0 + \left(\frac{\alpha_n A_1}{4}\right)(r^2 - h^2) + G\left(\frac{\theta_0}{4}(h^2 - r^2) + \frac{\beta_{nf}}{4}\left(\frac{3}{16}h^4 + \frac{r^4}{16} - \frac{r^2 h^2}{4}\right)\right) \qquad (21)$$

$$u(r,z) = 2\pi \cos 2\pi z\left(\frac{A_1 h r}{4} + \frac{G\beta_{nf}}{16}\left(\frac{r^3 h}{2} - \frac{3 r h^3}{2}\right)\right) \qquad (22)$$

And,

Where the nano fluid velocity components u and w in the direction of r and z respectively. The system (15-17) becomes in a closed form in a fixed frame when the volume flow rate takes the form

$$q = 2\int_0^h r\,(w(r,z))dr, \qquad (23)$$

The nano pressure gradient has the form

$$\frac{dP}{dz} = \frac{1}{\alpha_n}\left(\frac{8A_0}{h^2} - \frac{8q}{h^4} + G\left(\theta_0 - \frac{\beta_{nf}}{6}h^2\right)\right)$$

In the same way, on the basis of equations (17-19), the stream function becomes

$$\Psi(r,z) = \frac{A_0 r^2}{2} + \frac{A_1 \mu_1}{4}\left(\frac{r^4}{4} - \frac{r^2 h^2}{2}\right) + \frac{r^2 A_0}{2} - G\left(\frac{\theta_0}{4}\left(\frac{r^4}{4} - \frac{r^2 h^2}{2}\right) + \frac{\beta_{nf}}{4}\left(\frac{r^4 h^2}{16} - \frac{r^6}{96} - \frac{3 r^2 h^4}{32}\right)\right) \qquad (25)$$

Discussion and Results

The physical problem describes the $Al_2o_{3/}$ water nanofluid flow in a peristaltic vertical cylindrical tube. The mathematical model consists of mass, Navier-Stokes, heat, and volume rate equations (6-9) and (23) respectively is transformed to a non-dimensional one (11-14). The system (11-14) for long wavelengths is reduced to another system (15-17). The nano temperature (19), fluid velocity components (21-22), pressure gradient (24), and stream function (25) are obtained as a solution of the system (15-17) in analytical form.

Table (1): The physical parameters' values used in the present problem

	Density $\rho\,(kg/m^3)$	Thermal conductivity k(W/mK)	Specific heat c_p(J/kg K)
Water	**958.3**	**0.6857**	**4240**
Al_2O_3	**3700**	**46**	**880**

In the current problem, the range of void fraction for nanoparticles "$\boldsymbol{\varphi_p}$" = 0.01, 0.02 and 0.03. The physical values are illustrated in Table (1).

The nano temperature distribution $\boldsymbol{\theta_{nf}(r,z)}$ varies with "r" for different values of heat transfer parameter "$\boldsymbol{\beta}$"is shown in Fig. (2a). It is noted that, the temperature proportional directly with the heat transfer parameter values. The nano temperature distribution $\boldsymbol{\theta_{nf}(r,z)}$in terms of "r" for different values of amplitude "e" is shown in Fig. (2b). It is observed that, the temperature proportional inversely with amplitude values.

The fluid temperature distribution $\boldsymbol{\theta_{nf}(r,z)}$ varies with "r" for different values of void fraction for nanoparticles "$\boldsymbol{\varphi_p}$"is shown in Fig. (2c). It is noted that, the temperature proportional inversely with void fraction for nanoparticles $\boldsymbol{\varphi_p}$ values.

The nano fluid velocity w $\boldsymbol{(r,z)}$ in terms of "r" for different values of Grashof number "G_r" is shown in Fig (3a). It is noted that, the velocity proportional directly with the Grashof number values. The nano fluid velocity w $\boldsymbol{(r,z)}$ varies with "r" for different values of amplitude e is shown in Fig. (3b). It is noted that, the velocity proportional directly with amplitude values. The nano fluid velocity w$\boldsymbol{(r,z)}$ varies with "r" for different values of heat transfer parameter is shown in Fig. (3c). It is noted that, the velocity proportional directly with heat transfer parameter values.

The nano fluid velocity w $\boldsymbol{(r,z)}$ varies with "r" for different values of void fraction for nano particlesφ_p is shown in Fig. (3d). It is noted that, the fluid velocity proportional inversely with different values of void fraction for nano particles $\boldsymbol{\varphi_p}$ values.

The pressure gradient varies with "r" for different values Grashof number $\boldsymbol{G_r}$ is shown in Fig. (4a). It is noted that, the pressure gradient proportional directly with the Grashof number $\boldsymbol{G_r}$ values. The nano Pressure gradient of fluid varies with "r" for different values of amplitude e is shown in Fig. (4b). It is noted that, the pressure gradient proportional directly with amplitude e parameter values. The Pressure gradient varies with "r" for different values of heat transfer parameter $\boldsymbol{\beta}$ is shown in Fig. (4c). It is noted that, the pressure gradient proportional directly with the heat transfer parameter values. The Pressure gradient varies with “r” for different values of void fraction for nanoparticles φ_p is shown in Fig. (4d). It is noted that, the pressure gradient proportional inversely with the void fraction for nanoparticles φ_p.

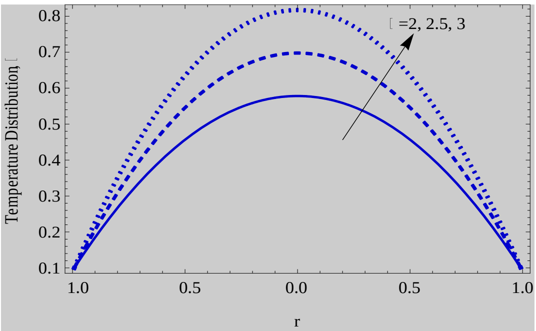

Fig. (2a): Temperature varies with amplitude

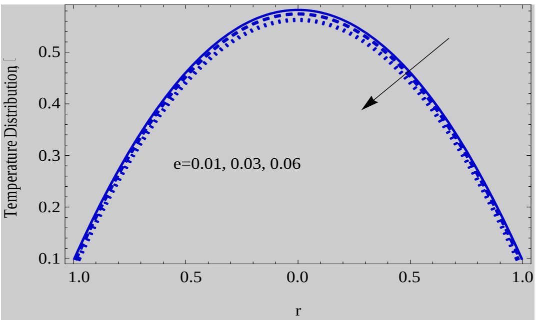

Fig. (2 b): Temperature varies with heat parameter "β”

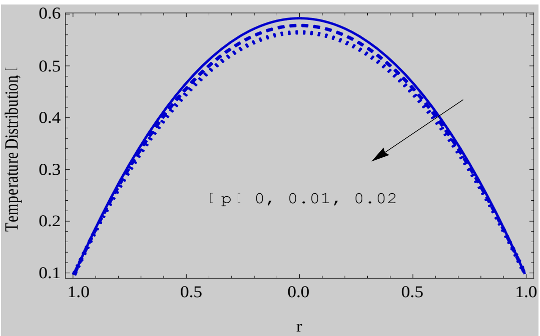

Fig. (2 c): The nano temperature $\theta(r,z)$ varies with "r" for three values of void fraction for nano particles φ_p

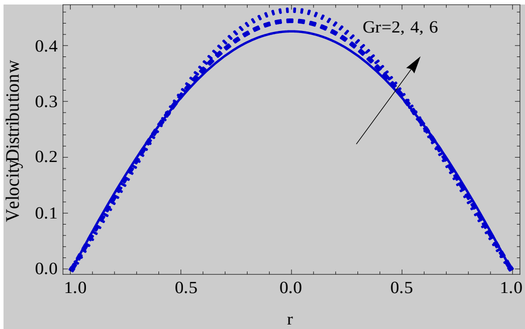

Fig. (3a): The nanofluid velocity varies with "r" for three values of and Grashof number “G_r"

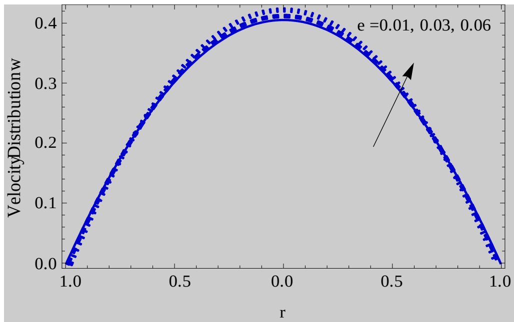

Fig. (3b): The nanofluid velocity varies with "r" for three values of amplitude “e”

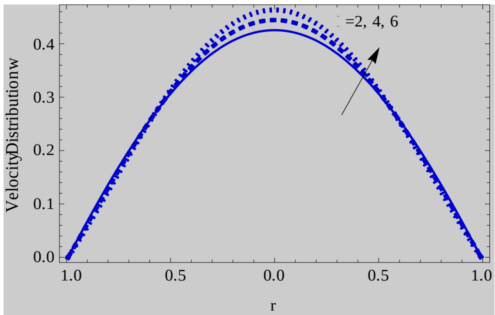

Fig. (3 c): The nanofluid velocity varies with "r" for three values of heat parameter "β"

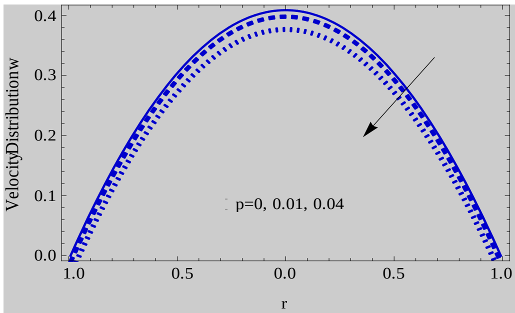

Fig. (3 d): The nanofluid velocity varies with "r" for three values void fraction for nano particles "φ_p"

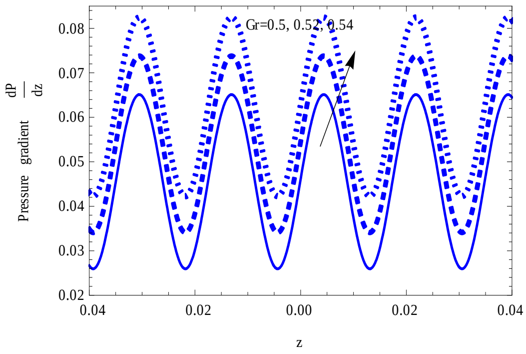

Fig. (4a): The nano pressure gradient varies with “z” for amplitude “e”

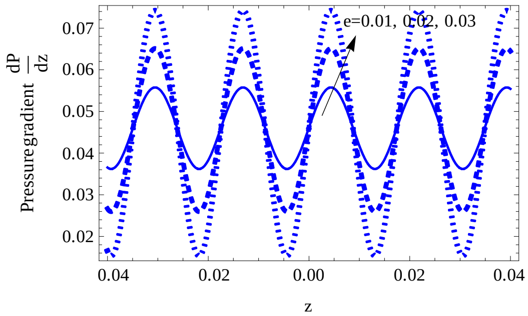

Fig. (4b): The nano pressure gradient varies with “z” for Grashof number **"G_r"**

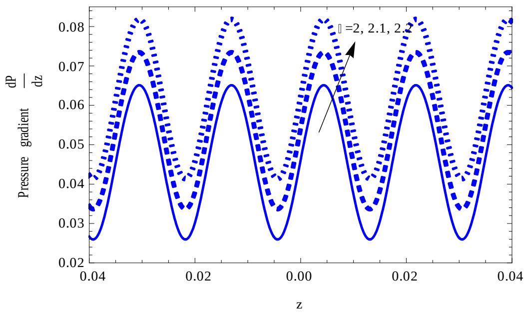

Fig. (4 c): The nano pressure gradient varies with **"z"** for three values of void fraction for nano particles**"φ_p"**

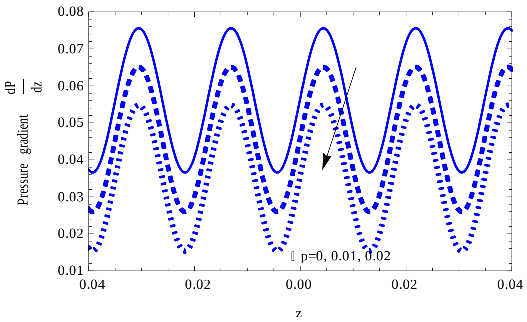

Fig. (4 d): The nano pressure gradient varies with **"z"** for three values of heat parameter **"β"**

Conclusions

The viscous incompressible Newtonian fluid flow in a peristaltic vertical cylindrical tube were analyzed in detail. The suspended nano particles ($A\,L_2O_3$) in the fluid flow affected behavior of physical problem. The nano temperature, fluid velocity and pressure gradient are affected by some physical parameters (amplitude ratio "e", Grashof number "G”, and heat transfer parameter **"β"** , void fraction for nano particles **"φ_p"** ; which are illustrated through the graphs.

The concluded remarks from the above discussion of results and figures are tabulated in the following points:

1. The nano temperature in the fluid flow increases with heat transfer parameter **"β"** . On contrary, temperature decrease with amplitude ratio "e" and void fraction for nano particles **"φ_p"**.
2. The nano fluid velocity increasing with amplitude ratio " e ", Grashof number and heat transfer parameter**"β"**.
3. The nano fluid velocity is proportional inversely with void fraction for nanoparticles **"φ_p"**.
4. The nano pressure gradient proportional directly with Grashof number, amplitude ratio e, and heat

transfer parameter "β" .values. In the same way, the pressure gradient proportional inversely with fraction for nanoparticles "φ_p".

5. The nano particles AL_2O_3 play a dominant factor of nano fluid flow.

6. The nanofluid temperature, fluid velocity and pressure gradient in a peristaltic vertical cylindrical tube prove the validity of the proposed model.

References

Bilicki, Z., Roman Kwidziński, Salim Ali Mohammadein (1996). Evaluation of the relaxation time of heat and mass exchange in the liquid-vapour bubble flow. *Int. j. heat and mass transfer*, 39(4), 753-759.

Brinkman, H.C. (1952), The viscosity of concentrated suspensions and solutions. *J.Chem.Phys*., 20: 571-581.

Landau, L. D. and E. M. Lifshitz (1987). Fluid Mechanics, **6:** Second edition "Nauka", Moscow.

Mohammadein, S. A., Abu-Nab, A. K. and G. A. Shalaby (2017). The behavior of vapour bubbles inside a vertical cylindrical Tube under the effect of peristaltic motion with two-phase density flow and heat transfer. *J. Nanofluids*, 6: 1–6.

Mohammadein, S. A. and A. K. Abu-Nab (2019). Peristaltic flow of a Newtonian fluid through a porous medium surrounded vapour bubble in a curved channel. *J. Nanofluids*, 8: 651–656.

Elbendary, A. A., Mohammadein, S. A., FA El Hussiny, SM Elkholy, MS Ali (2022). Peristaltic Flow for A Mixture of Blood and Gas Bubbles in the Vertical Inferior Mesenteric Vein. *Appl. Math*., 16 (6): 935-942.

Mohammadein, S. A., Atef A. Hemada, Magdy A. El-Tantawy, Muhammad A. Baghagho and Maha S. Ali (2023). The Effect of Peristaltic Motion on Nano Fluid (CuO/Water) Inside a Vertical Cylindrical Tube. DJS, 46(3):99-110

Mohammadein, S. A. (2020) New Treatment of Fluid Mechanics with heat and Mass Transfer: Theory of Diffusion. *Appl. Math & Info Sci. Lett. J*., 8(1):1-6

Mohammadein, S. A.., R.G. El-Rab and Maha S. Ali (2021). The Simplest Analytical Solution of waterNavier-Stokes Equations. *Info. Sci Lett*.,10 (2):159-165.

Mohammadein, S. A., Ali S. Ali and Maha S. Ali (2022). The Analytical and simplest Resolution of Linear Navier-Stokes Equations. *Appl. Math & Info Sci. Lett. J*., 16(6): 1043-1048.

الإنسياب التمعجي لمائع نانوي($Water/Al_2o_3$) غير قابل للإنضغاط تحت تأثير إضافة/إمتصاص الحرارة داخل الأنبوب الإسطواني الرأسى

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الإنسياب التمعجي لبعض السوائل في الأنابيب الأسطوانية الرأسية والأفقية تمت دراستة بشكل غير مغلق من ناحية النموذج الرياضى لكثير من الباحثين. في هذا البحث تمت دراسة تدفق المائع النانوي ($Water/Al_2o_3$) في الأنبوب الأسطواني الرأسى التمعجي تحت تأثير الجاذبية وثبوت مقدار الإضافة والإمتصاص لكمية الحرارة. تمثل معادلات الكتلة ونافير ستوكس والحرارة ومعدل التدفق الحجمي وصفا للنموذج الرياضي فى حالة الأطوال الموجية الطويلة فقط. وأن نظام المعادلات يمثل شكلاً مغلقًا للبارامترات التابعة. تم حل النموذج تحليليا بدلالة توزيعات درجة حرارة المائع النانوية وسرعته وتدرج الضغط مع الأخذ فى الإعتبار وجود الجسيمات النانوية العالقة في الماء. كذلك تم الحصول على دالة الإنسياب في مستوى الخليط من السائل والجسيمات النانوية. والنتائج تبن أن توزيع درجة حرارة النانوى في الأنبوب التمعجي يتأثر بقيم مختلفة لنسبة السعة وبارامبتر مصدر التسخين. ولوحظ أن تدرج الضغط وسرعة مائع النانو يزدادان مع تغير قيم أرقام جراشوف ومعدل تدفق الحجم وبارامبتر مصدر التسخين وتتناقصان مع مقدار الجزء الفراغى لجزيئات النانو. النتائج التي تم الحصول عليها في الشكل المغلق للنموذج الرياضى تثبت صحة النموذج الحالي المقترح.