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Research Article

MATHEMATICS

New Treatment of Wave Solutions for the Nonlinear Burgers Equation

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Nonlinear Burger equation. Mohammadein concept. Linear velocity operator, Linear Burger equation, Incompressible fluid, viscous, Newtonian, fluid flow

ABSTRACT

The nonlinear partial differential equations face many obstacles due to the nonlinear terms. The numerical and approximate solutions are only possible to be obtained. Most of the previous problems which were described by the nonlinear Navier-Stokes, Burger equations were solved approximately for long wavelengths and low Reynolds number. In this paper, the unsteady nonlinear Burgers equation is converted to the linear diffusion equation based on the concept of linear velocity operator for the first time. The obstacle in the nonlinear term is solved in the nonlinear Burgers equation on the basis of Mohammadein concept. The Burgers equation is studied in one dimension. The simplest analytical solution of linear Burger equation is obtained by Picard method. The analytical and simplest solution is obtained in terms of fluid velocity in normal scale. The fluid velocity is affected by fluid viscosity, time and wavelengths. The calculated results introduced the validity of the present model.

Introduction

The nonlinear partial differential equations represent many physical motions and behavior of particles in fluid mechanics with heat and mass transfer **(Landau and Lifshitz, 1987)**. A systematic literature review of Burgers' equation with recent advances is introduced **(Mayur, 2018)**. Some recent researches on the motion of fluids are performed **(Bateman, 1915)**. Mathematical examples illustrating relations occurring in the theory of turbulent fluid motion **(Burgers, 1939)**. There are some numerical methods of Burger equation. One-dimensional coupled Burgers' equation and its numerical solution by an implicit logarithmic finite-difference method **(Vineet et al., 2014)**. A table of solutions of the one-dimensional Burgers equation, quarterly of applied mathematics **(Benton and Platzman, 1972)**.

There are many applications in many fields. The periodic wave shock solutions of Burgers equations are obtained **(Bendaas, 2018)**. The Cole-Hopf transformation is used for solving Rayleigh-Plesset equation in bubble dynamics **(Abu-Bakr and Abourabia, 2018)**.

Recently, the non-linear of Navier-Stokes, Burgers, Korteweg-De Vries equations are transformed to linear partial differential equations. Moreover, the linear heat, mass, concentration is transformed to simplest linear equations **(Mohammadein, 2020)**.
The simplest analytical solution of Navier-Stokes equations is obtained for a first time **(Mohammadein et al., 2021)**. The analytical and simplest resolution of linear Navier-Stokes equations as closed system is obtained in two dimensions for a first time in international journals **(Mohammadein et al., 2022)**. The investigation of an incompressible viscous Newtonian fluids flow in three-dimensions using linear Navier-Stokes equations is discussed in detail **(Ali et al., 2023).**

On the basis of Euler concept of fluid state in fluid mechanics, the definition of total derivative for any dependent function "f "has the form

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + (\underline{\hat{v}}.\underline{\nabla})f \,, \qquad (1)$$

where $\underline{\hat{v}}$ is the linear operator of fluid velocity on the basis of Mohammadein theory **(Mohammadein, 2020)** becomes

$$\underline{\hat{v}} = -M^*\underline{\nabla} \,, \qquad (2)$$

where M^* is Mohammadein parameter, which takes different values in fluid mechanics and heat mass transfer. On the basis of equation (2), the equations (1) become in the form:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} - (M^* \underline{\nabla}^2) f \,, \qquad (3)$$

then, the fluid acceleration takes the form

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} - nv\underline{\nabla}^2\underline{v} \,, \qquad (4)$$

where $M^* = nv$ for Burger's equation.

The incompressible viscous Newtonian fluid flow, which is described by nonlinear Burger's equation has the form

$$\frac{\partial u}{\partial t} + nu\frac{\partial u}{\partial x} = vu_{xx} \qquad (5)$$

where n is constant, v is kinematic viscosity.

In this paper, the nonlinear Burgers equation is converted to a simple linear equation on the basis of **(Mohammadein, 2020)**. The linear Burgers equation is solved by Picard method in analytical way **(Ali, et al., 2023)**. The solutions of fluid velocity are obtained in terms of kinematic viscosity, time and parameter "n".

Analysis

The present problem represents a solution for different physical phoneme in fluid mechanics. In one dimension, the nonlinear Burgers equation is

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} - \left(nv\frac{\partial^2}{\partial x^2}\right)u \,. \qquad (6)$$

In another form:

$$\frac{\partial u}{\partial t} - \left(nv\frac{\partial^2}{\partial x^2}\right)u = vu_{xx} . \qquad (7)$$

Then, using Mohammadein theory **(Mohammadein, 2020)**, the above Burgers equation is converted to the linear equation in one dimensional cartesian coordinates in the form

$$\frac{\partial u}{\partial t} = v(n+1)u_{xx} \,. \qquad (8)$$

Applying Picard method as in Appendix, then the analytical solution of above equation (8) has the following form

$$u(x,t) = A_1 e^{[(n+1)v\,t c_1^2 - c_1 x]} . \qquad (9)$$

To find the constants A_1 and c_1 on the basis of the following initial and boundary conditions

I.C.: $u(x,0) = e^{-c_1 x} \rightarrow A_1 = 1$.

B.C $(L_1, 0) = q_1$, $u(L_2, 0) = q_2$

$$\rightarrow c_1 = \frac{1}{L_2 - L_1} log\left(\frac{q_2}{q_1}\right) \qquad (10)$$

The solution (9) is dependent on the values of constants A_1 and c_1.

Discussion of Results

The incompressible Newtonian fluid flow in one dimension Burger equation is studied. The nonlinear Burgers equation on the basis of "New Treatment theory **(Mohammadein 2020)** is given by a linear equation (8) for a first time. The analytical solution is obtained by equation (9) for a first time for a linear Burger equation (8). The solution (9) is drawing graphically for three different values of physical parameters. The present graphs are performed for kinematic viscosity" υ" and time "t" when $n \leq -2$. On contrary, the fluid velocity is calculated for constant values of time "t" and kinematic viscosity υ for all positive and negative values of parameter "n".

The current calculations for the dimensional equation consider:

$L_1 = 0.1, L_2 = 0.6, c_1 = 0.89$

The fluid velocity in terms of distance x for different values of kinematic viscosity is shown in Fig. (1). It is observed that the fluid velocity is proportional inversely with kinematic viscosity. The fluid velocity in terms of distance x varies with different values of time is shown in Fig. (2). It is observed that the fluid velocity is proportional inversely with time

The fluid velocity in terms of distance x for different values of positive and negative parameter n are shown in Figs. (3 and 4). It is observed that the fluid velocity increases with all positive and negative values of parameter "n".

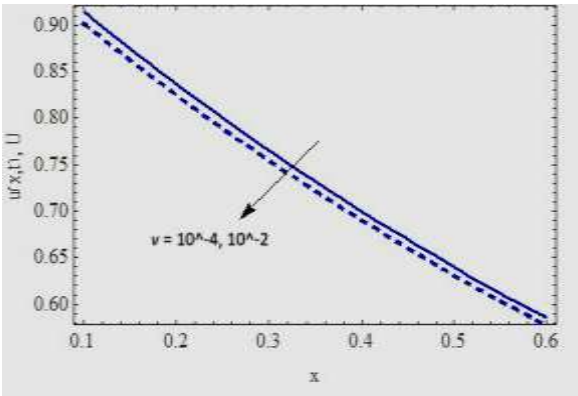

Fig. (1): The fluid velocity in terms of distance x for different values of kinematic viscosity

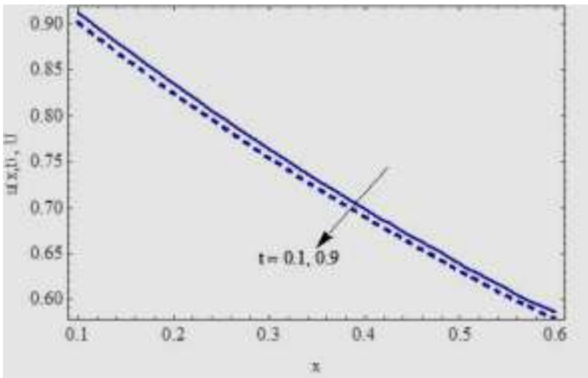

Fig. (2): The fluid velocity in terms of distance x for different values of time

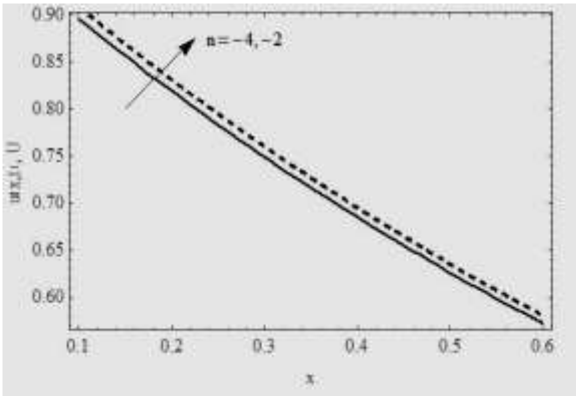

Fig. (3): The fluid velocity in terms of distance x for negative values of parameter n

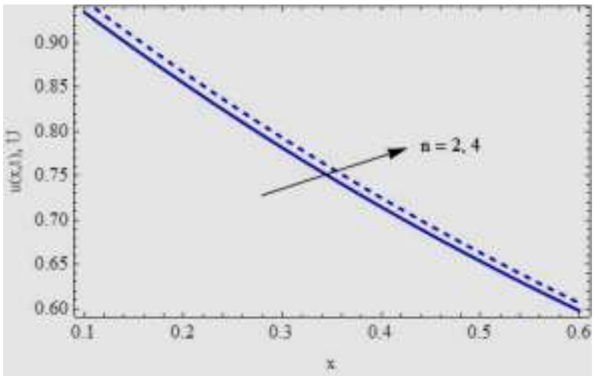

Fig. (4): The fluid velocity in terms of distance "x" for positive values of parameter "n"

Conclusions

The Burger equation is considered as a special case of Navier-Stokes equation. The nonlinear Burgers equation (3) is converted to a linear partial differential equation (8) on the basis of Mohammadein concept **(Mohammadein, 2020)**.

The equation (9) represents the analytical solution of linear Burger's equation after converting. The solution (9) is drawing graphically for some different values of physical parameters. The obtained results and graphs are concluded in the following remarks:

1. Burger's equation is transformed for nonlinear to linear partial differential equation for the first time by a simplest method.

2. Burger's equation is considered as a special case from Navier-Stokes where the parameter "n" has a sensitive effect on the results which agree with the physical meaning.

3. The fluid velocity is proportional inversely with kinematic viscosity "υ" and time "t" only when $n \leq -2$ but it cannot be verified physically at positive values of "n".

4. On contrary, the fluid velocity is proportional directly with all positive and negative values of "n" at constant values of time and kinematic viscosity.

5. The parameter "n" plays a dominant parameter to determination of fluid diffusion amount.

6. The obtained results prove validity of the suggested physical and mathematical model of the problem.

Appendix

Picard method

The Picard method is applied for solving the following linear Burger equation:

$$\frac{\partial u}{\partial t} = \nu(n+1)u_{xx} \tag{A1}$$

$$u(x,0) = A_1 e^{-c_1 x} \tag{A2}$$

$$u_{N+1} = u_0 + \nu(n+1)\int_0^t (u_N)_{xx} dt\ . \tag{A3}$$

where N=0,1,2,3….

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معالجة جديدة للحلول الموجية لمعادلة برجر غير الخطية

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نلاحظ أن المعادلات التفاضلية الجزئية غير الخطية تواجه العديد من العقبات الناجمة عن الحدود غير الخطية بينما يظل من الممكن غالبا الوصول إلى حلول عددية أو تقريبية لمثل هذه المسائل المعقدة. هكذا نجد أن معظم المسائل التي سبق وصفها بمعادلات نافيير ستوكس غير الخطية وبرجر تم حلها تقريبيا في حالات خاصة حيث يتم التعامل مع أطوال موجية طويلة وأعداد رينولدز صغيرة القيمة. ونلاحظ أن "معادلة برجر أحادية البعد" ظهرت أولًا بواسطة باتمان ثم كحالة خاصة لنماذج الاضطراب التي وضعها برجر. في الماضي تم إستخدام بعض التحويلات مثل هوف وكول كما استخدمت بعض الطرق العددية. هناك العديد من التطبيقات في العديد من المجالات مثل موجات الصدمة الدورية ومعادلة برجر الكسرية في الصوتيات غير الخطية بل تتجاوز التطبيقات ميكانيكا الموائع إلى بعض المجالات المتقدمة مثل علم الكوزمولوجي والحاسوب الكمي.

في هذا البحث تم تحويل معادلة برجر غير الخطية غير المستقرة إلى معادلة الانتشار الخطي بناءً على مفهوم مؤثر السرعة الخطية. ولأول مرة يتم حل العائق المعتاد المتمثل في الحد غير الخطي في معادلة برجر غير الخطية. للتبسيط تم حل المعادلة في بعد واحد. الحل التحليلي المبسط أستند على طريقة بيكارد. بالإضافة إلى ما سبق فإن الحل الناتج يعتمد على سرعة المائع كما أن سرعة المائع تعتمد على لزوجة المائع والزمن والأطوال الموجية. تمت دراسة هذه المسألة في المقياس المعتاد وليس لأجل حالات خاصة استثنائية وقدمت الحسابات والرسومات برهانا على صحة النموذج الرياضي المقترح.