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NONLINEAR INSTABILITY OF FERROFLUIDS IN POROUS MEDIA UNDER A HORIZONTAL MAGNETIC FIELD

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ABSTRACT

The nonlinear instability analysis of the free surface of two weak viscous magnetic fluids, subjected to vertical vibrations and a horizontal magnetic field, has been examined in porous media. The two fluids are immiscible in all properties. Both have finite-thickness, homogeneous, and incompressible fluids. Although the motions are assumed to be irrotational, weak viscous effects are included in the boundary conditions of the normal stress tensor balance. The influence of both surface tension and gravity force is also considered. The method of multiple scale perturbations is used to obtain a dispersion relation for the linear theory and a Ginzburg-Landau equation for the nonlinear theory, describing the behaviour of the system. There is also the obtaining of a nonlinear diffusion equation, describing the evolution of the wave packets, near the marginal state. Further, the nonlinear Schrodinger equation is obtained when the effect of both the viscosity and Darcy's coefficients are neglected. The stability conditions are discussed and the interplay between the applied magnetic field and several other factors in determining the interface behaviour is analyzed. Stability analysis and numerical calculations are used to describe linear and nonlinear stages of the interface evolution. The numerical calculations indicate the existence of more than a new region of stability and instability due to the nonlinear effects. In the linear theory, it is found that the horizontal magnetic field decreases as the wave number increases. This means that the magnetic field has a stabilizing influence on the wave motion. While the viscosity and Darcy's coefficients have a destabilizing effect. In the nonlinear theory, it is found that these parameters have an important role in the stability criterion of the problem.

INTRODUCTION

Ferrofluids, also known as magnetic fluids, are composed of three fundamental components: magnetic particles, surfactant and base oil. The study of various phenomena of ferrofluids is of fundamental interest and importance with respect to the variety of applications.

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Therefore, extensive investigations of ferrohydrodynamics have been conducted and reported [1].

These fluids are stable colloidal systems containing single domain ferro-or ferri-magnetic particles [2]. They respond readily to magnetic fields and have a saturation magnetization which can be as high as 20 % of the solid magnetic component. A large number of applications for these fluids appears possible, including novel energy conversion schemes, levitation devices, magneto-optical devices, novel zero-leakage rotary shaft seals and pressure seals. In these developments, an understanding of the fundamental magnetic fluid dynamics is essential. In systems of stable homogeneous magnetic fluids, surface interactions are particularly important. A number of striking interfacial phenomena are exhibited by the magnetic fluids in response to applied magnetic fields.

A magnetic ferrofluid is an interdisciplinary topic having inherent interest of a physical and mathematical nature with applications in tribology, separations science, instrumentation, information display, printing, medicine, and other fields. Many experimental results confirmed that colloidal particles in ferrofluids coagulate and form chain clusters as a result of their mutual interaction; this process being enhanced in the presence of a magnetic field. The chain formation process, together with the reorientation of individual particles in the presence of a magnetic field, are responsible for the anisotropy of the physical properties of the magnetic fluids. For example, the sound velocity and the acoustic attenuation coefficient in magnetic fluids are dependent on the angle between the sound propagation direction and the external magnetic field [3]. The reader can find much more information about these fascinating complex fluids in Rosensweig's classical book[1].

On the other hand, the moving interface between two superposed fluids in a porous medium or in a Hele- Shaw cell is unstable forming penetrating fingers, when the more viscous fluid, is pushed by the less viscous fluid [4]. Previously reported analysis has shown that the use of a magnetizable fluid layer can stabilize the interface if a uniform magnetic field is applied tangentially to the interface [1]. A magnetic field component perpendicular to the interface is always destabilizing. In the absence of the magnetic field, Mikaelian[5] studied the effect of viscosity with the surface tension on Rayleigh- Taylor instability for two finite- thickness. He obtained numerical results, by solving the dispersion relation, which determines the growth rate as a function of the physical parameters of the system.

Porous media theories play an important role in many branches of engineering, including material science, the petroleum industry, chemical engineering, and soil mechanics , as well as, biomechanics. The flow through a porous medium has gained considerable interest in recent years, particularly, among geophysical fluid dynamicists [6]. A porous medium is a matter, which contains a number of small holes distributed throughout the matter. Flows through porous medium occur in filtration of fluids. Fluid flow in porous media is an important subject in hydrology and is of vital interest to the petroleum industry. Henry P.G.Darcy gave the law governing seepage flow of a homogeneous fluid in a homogeneous and isotropic porous medium in his Paris Treatise (1856). This law is valid only for very slow flows. The gross effect, as the fluid slowly percolates through the porous rock, is represented by Darcy's law, which describes the flow of an incompressible fluid through a homogeneous and isotropic porous medium. El-Dib and Ghaly [7] studied the unsteady flow of a viscous fluid through a porous medium, by an infinite vertical surface, by taking into account the pressure of the free connective currents, when there is a variable suction at the surface.Nonlinear waves occurring in multi-phase flow in porous media have a rich structure. In contrast to two-phase flow, the structure is often very sensitive to diffusive terms, i.e. capillary pressure.

The study of viscous flow in a Hele- Shaw cell is of interest for both scientific and practical reasons. On the scientific level, the influence of spatial curvature on hydrodynamic flow is a matter of fundamental interest. It also provides a simple mathematical model to describe more general situations involving the filling of a thin cavity between two walls of a given shape with fluid. On the practical level, it may have applications in a number of industrial, manufacturing processes, ranging through pressure moulding of molten metals and polymer materials, and formation of coating defects in drying paint thin films [1,6].

Work in nonlinear stability theory has received greater interest in recent years. The study of the nonlinear interfacial instability has received a considerable number of contributions [8-13]. Hasimoto and Ono [8] derived a nonlinear Schrödinger equation describing the finite amplitude wave packets on a fluid surface with the use of a multiple scale method. Nayfeh [9] carried out the nonlinear analysis for the Rayleigh-Taylor instability and obtained two nonlinear Schrödinger equations for the progressive and stationary waves. Elhefnawy [12] used the method of multiple scales to study the nonlinear dynamics of normal field instability in electrohydrodynamics. He derived a nonlinear damped Klein-Gordon equation and formulated the Melnikov function to show that if the ratio of forcing to dissipation is sufficiently small, then there exists transverse homoclinic orbits resulting in chaotic behaviour. Moatimid [13] deduced the well-known nonlinear Schrödinger equation with complex coefficients, describing the evolution of the wave train of a magnetic fluid jet through porous media. He found that the porous media have a destabilizing influence.This influence is enhanced when the Darcy's coefficients are different.

However, the uniform horizontal magnetic field only stabilizes those waves propagation in the direction along the magnetic field, having no effect on transverse waves. Previous works by others with analogous inviscid systems has shown that the non-uniform magnetic field can stabilize all waves with a lower required value of magnetic field. This occurs because the interface feels a restoring perturbation force even without distorting but by simply moving through the non- uniform magnetic field. This paper examines the nonlinear effect of a horizontal magnetic field on stabilization or destabilization of fluid flows through porous media.

The instability in a porous medium of a plane interface between two weak viscous magnetic fluids may be of interest in geophysics and biomechanics and is therefore studied in the present paper. The effect of a surface tension, a constant acceleration and a horizontal magnetic field, being relevant for geophysics, are also considered.

The aim of the work presented here, is to study the impact of nonlinear weak viscous effect of interfacial instabilities of two superposed magnetic fluids. In addition, we intended to examine the porous material effects with the presence of viscous force. The system is stressed by a horizontal magnetic field. The linearized problem has been demonstrated, without magnetic field and porosity, by Joseph et al. [14] and Pan et al. [15]. In the following, we shall first formulate the general interfacial problem. Then the nonlinear analysis, using a multiple scale expansion, is to be carried out. The stability criteria are discussed both theoretically and numerically for the different cases, and the stability diagrams are drawn.

2. SYSTEM AND EQUATIONS OF THE PROBLEM

2.1 BASIC EQUATIONS

The system under consideration, is composed of two incompressible magnetic fluids separated by the plane $z = 0$. Each fluid is of infinite horizontal extent. We take the origin o at the mean level of the interface, and the axis oz pointing vertically upwards into the upper fluid. Let the two fluids be confined between rigid horizontal planes $z = -h_1$ (the lower boundary)and $z = h_2$ (the upper one). The two fluids are influenced by a uniform magnetic field H_0 acting along the positive x- direction, where the axis ox is the mean level of the wave. The two media are considered as porous. Darcy's equation is a macroscopic equation which describes the flow of an incompressible Newtonian fluid of viscosity μ through a homogeneous and isotropic porous medium of permeability K. In this equation the usual viscous term is replaced by the resistance term $(-\eta \underline{v})$, where $\underline{v}$ is the filter velocity of the fluid and η is the Darcy's coefficient which depends on the ratio of the fluid viscosity to the flow permeability through the voids. The system is also subjected to a gravitational force g in the negative z -direction.

Thus, the system is governed by the continuity and the Darcy's equations, of an incompressible fluid through a porous medium where the porosity is considered as a unity,

$$\nabla . \underline{v} = 0 \, , \tag{2.1}$$

$$\rho [\frac{\partial \underline{v}}{\partial t} + (\underline{v} . \nabla) \underline{v}] = -\nabla p - \rho g \underline{e}_z - \eta \underline{v} \, , \tag{2.2}$$

where ρ, p and $\underline{e}_z$ are the fluid density, the pressure and the unit vector in the z- direction. Since the viscous force $(\mu \nabla^2 \underline{v})$ is dropped from equation(2.2), the viscosity contribution will be given in the stress tensor[1,7]

$$\sigma_{ik} = -(p + \tilde{\mu} H^2/2) \delta_{ik} + H_i B_k + \mu \; (\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}) \, , \tag{2.3}$$

where $\tilde{\mu}$, H, B and δ_{ik} are the magnetic permeability, the magnetic field, the magnetic induction and the Kronecker's delta.

We now follow the analysis of Joseph et al.[14] with two uniform parallel flows along the x - axis. The upper fluid has density ρ_2, viscosity μ_2, Darcian coefficient η_2, and magnetic permeability $\tilde{\mu}_2$, while the lower one has ρ_1, μ_1, η_1, $\tilde{\mu}_1$. A surface tension exists between the two fluids and denoted by σ. We assume that the small perturbations in each fluid are irrotational. The dynamics of this two - fluid problem can be analyzed using viscous potential flow [15]. Then, the fluid velocity is given as the gradient of a potential, i.e. $\underline{v} = \nabla\Phi$, and for incompressible fluids, the two velocity potentials $\Phi_1(x, z, t)$ and $\Phi_2(x, z, t)$ satisfy the following Laplace's equations:

$$\nabla^2\Phi_1 = 0 \qquad for \qquad -h_1 < z < \gamma(x,t), \tag{2.4}$$

$$\nabla^2\Phi_2 = 0 \qquad for \qquad \gamma(x,t) < z < h_2, \tag{2.5}$$

where $\gamma(x, t)$ denotes the elevation of the interface at time t.

In the case of a magneto- quasi- static system with negligible displacement current, Maxwell's equations in the absence of free currents reduce to Gauss' law and Ampére's law (no currents),

$$\nabla.\underline{B} = 0 \qquad and \qquad \nabla \wedge \underline{H} = \underline{0}, \tag{2.6}$$

where$\underline{B} = \tilde{\mu}\underline{H}$ is the magnetic induction vector.

From Ampére's law, the magnetic field can be expressed in terms of a magnetic scalar potential $\Psi(x, z, t)$ in each of the regions occupied by the fluids, i.e.

$$\underline{H}_j = H_0\underline{e}_x - \nabla\Psi_j \ , \qquad j = 1, 2, \tag{2.7}$$

where $\underline{e}_x$ is the unit vector along the x- direction and the subscripts 1 and 2 refer to quantities in the lower and upper fluids, respectively.

Then, combining the latter equation(2.7) with Gauss' law, considering $\tilde{\mu}$ is a constant, one finds that the magnetic scalar potentials, also, obey Laplace's equations:

$$\nabla^2 \Psi_1 = 0 \qquad for \qquad -h_1 < z < \gamma(x,t), \tag{2.8}$$

$$\nabla^2 \Psi_2 = 0 \qquad for \qquad \gamma(x,t) < z < h_2. \tag{2.9}$$

To complete the formulation of the problem, we must define the surface geometry and supplement the magnetic equations with the corresponding boundary conditions. The interface is represented by the expression

$$F(x,z,t) = z - \gamma(x,t) = 0 \ ,$$

for which the outward normal vector is written as

$$\underline{n} = \frac{\nabla F}{|\nabla F|} = [1 + (\frac{\partial \gamma}{\partial x})^2]^{-\frac{1}{2}} \qquad (-\frac{\partial \gamma}{\partial x}, 0, 1). \tag{2.10}$$

2.2 NONLINEAR BOUNDARY CONDITIONS

The solutions for both, Φ_j and Ψ_j , j=1,2, have to satisfy the following revelant boundary conditions for our configuration [7,9,10,11,13,14]:

(i) **On the rigid boundaries** $z = -h_1$ and $z = h_2$,

(1)the normal fluid velocities vanish on both the bottom and the top boundaries, i.e.

$$\frac{\partial \Phi_j}{\partial z} = 0 \ , \ on \qquad z = (-1)^j h_j \ , \qquad j = 1,2 \tag{2.11}$$

(2)the tangential components of the magnetic field vanish on these boundaries, i.e.

$$\frac{\partial \Psi_j}{\partial x} = 0 \ , \ on \qquad z = (-1)^j h_j \ , \qquad j = 1,2 \tag{2.12}$$

(ii) **On the free interface** $z = \gamma(x,t)$, (1)the normal components of the velocity potential are compatible with the assumed form of the boundary. This is called kinematical boundary condition , which gives

$$\frac{\partial\gamma}{\partial t}+\frac{\partial\gamma}{\partial x}\,\frac{\partial\Phi_j}{\partial x}=\frac{\partial\Phi_j}{\partial z}\,,\qquad\qquad j=1,2\,, \tag{2.13}$$

(2)the normal components of the magnetic induction vector are equal, since we have assumed that there is no free currents at the interface. This can be written as $\underline{n}\cdot[|\underline{B}|]=0$, or

$$[|\tilde{\mu}\frac{\partial\Psi}{\partial z}|]+H_0(\tilde{\mu}_2-\tilde{\mu}_1)\frac{\partial\gamma}{\partial x}=\frac{\partial\gamma}{\partial x}\,[|\tilde{\mu}\frac{\partial\Psi}{\partial x}|]\,, \tag{2.14}$$

where $[|\quad|]$ indicates the jump or difference across the interface, i.e. $[|\ X\ |]=X_2-X_1$,

(3)the tangential components of the magnetic field are equal at the interface, $\underline{n}\wedge[|\underline{H}|]=\underline{0}$, or

$$[|\frac{\partial\Psi}{\partial x}|]++\frac{\partial\gamma}{\partial x}\,[|\frac{\partial\Psi}{\partial z}|]=0\,, \tag{2.15}$$

(4)finally, the continuity of the normal stress at the perturbed surface $z=\gamma(x,t)$ is

$$\underline{n}\cdot[|\underline{\Pi}|]=\sigma\ \nabla\cdot\underline{n}\,,$$

or

$$(\frac{\partial\gamma}{\partial x})^2[|\sigma_{11}|]-2\frac{\partial\gamma}{\partial x}[|\sigma_{13}|]+[|\sigma_{33}|]=\sigma\frac{\partial^2\gamma}{\partial x^2}[1+(\frac{\partial\gamma}{\partial x})^2]^{-1/2}\,, \tag{2.16}$$

where $\underline{\Pi}$ is the force vector acting on the interface, given by

$$\underline{\Pi}=\begin{bmatrix}\sigma_{11}&\sigma_{31}\\ \sigma_{13}&\sigma_{33}\end{bmatrix}\begin{bmatrix}n_1\\ n_3\end{bmatrix}.$$

The continuity of the tangential components of the stress tensor is identically zero because of the continuity of the magnetic inductions and fields in (2.14) and (2.15), respectively.

2.3 PERTURBATION ANALYSIS

To describe the nonlinear interactions of small but finite amplitude, we use the method of multiple scales formulated by Nayfeh[9]. We 'll find solutions in the neighbourhood of the neutral curve where the linear growth rate of instability is small. We intend to describe the mode which is being modulated slowly in time and space. We, therefore, introduce spatial and temporal scales:

$$x_m = \epsilon^m x \quad , t_m = \epsilon^m t \qquad (m = 0, 1, 2) \ , \tag{2.17}$$

where ϵ represents a small parameter characterizing the steepness ratio of the wave.

The various physical quantities can now be expanded in the form:

$$f(x, z, t) = \sum_{m=1}^{3} \epsilon^m f_m(x_0, x_1, x_2, z, t_0, t_1, t_2) + O(\epsilon^4), \tag{2.18}$$

where f can be any one of the physical quantities Φ_j , Ψ_j and $\gamma(x, t)$. While writing the expansion for γ, it will be noted that γ depends only on x and t and not on z. Also, for the derivatives, we write

$$\frac{\partial}{\partial \beta} = \sum_{m=0}^{2} \epsilon^m \frac{\partial}{\partial \beta_m} + O(\epsilon^3), \tag{2.19}$$

where β is any of the variables x or t.

To evaluate the boundary conditions on the interface $z = \gamma(x, t)$, we use the Maclaurin series expansions at $z = 0$ for the quantities involved. The approximate solutions of equations (2.4) , (2.5) , (2.8) and (2.9), with the nonlinear boundary conditions (2.13) - (2.16) are derived with use of equations (2.17) - (2.19) and equating the coefficients of equal powers of ϵ. The hierarchy of equations so obtained for each order can be solved with the knowledge of solutions for the previous orders. The procedure is straightforward but lengthy and it will not be included here. The details are available from the authors (and are outlined by [9]).

3. THE LINEAR THEORY

The solution of the first - order or linear problem, in the form of a travelling wave, is

$$\gamma_1 = A \ e^{i\theta} + c.c., \tag{3.1}$$

$$\Phi_{11} = -\frac{i\omega \cosh k(z+h_1)}{k \sinh kh_1} A \ e^{i\theta} + c.c., \tag{3.2}$$

$$\Phi_{21} = \frac{i\omega \cosh k(z-h_2)}{k \sinh kh_2} A \ e^{i\theta} + c.c., \tag{3.3}$$

$$\Psi_{11} = \frac{iH_0(\tilde{\mu}_2 - \tilde{\mu}_1)\sinh k(z+h_1)}{\tilde{\mu}(k)\sinh kh_1} A \ e^{i\theta} + c.c., \tag{3.4}$$

$$\Psi_{21} = -\frac{iH_0(\tilde{\mu}_2 - \tilde{\mu}_1)\sinh k(z-h_2)}{\tilde{\mu}(k)\sinh kh_2} A \ e^{i\theta} + c.c., \tag{3.5}$$

where $\tilde{\mu}(k) = \tilde{\mu}_2 \coth(kh_2) + \tilde{\mu}_1 \coth(kh_1)$, $\theta = kx_0 - \omega t_0$ and (x_0, t_0) is the lowest scale. The amplitude A of the progressive wave is a function of the faster scales (x_1, x_2, t_1, t_2) and will be determined later from the solvability conditions , c.c. denotes the complex conjugate of all the preceding terms, i $(= \sqrt{-1})$ is the imaginary number, k is the wave number, ω is the frequency of the wave which can be a complex number. An imaginary part for ω indicates a disturbance which either grows with time (instability) or decays with time (stability), depending on whether this imaginary part is positive or negative, respectively. If $\omega = 0$ the disturbance is neutrally stable.

The non- trivial solutions (3.1) - (3.5) satisfy the dispersion relation, with frequency ω and wave number k, given by

$$D(\omega, k) = g(\rho_1 - \rho_2) + kH_0^2\delta_0(k) + \sigma k^2 - \frac{i\omega}{k}[\eta_1 \coth kh_1 + \eta_2 \coth kh_2$$

$$+2k^2(\mu_1 \coth kh_1 + \mu_2 \coth kh_2)] - \frac{\omega^2}{k}(\rho_1 \coth kh_1 + \rho_2 \coth kh_2), \tag{3.6}$$

where $\delta_0(k) = (\tilde{\mu}_2 - \tilde{\mu}_1)^2/\tilde{\mu}(k)$.

It may be noted that, the results obtained by Mikaelian [5] and Pan et al. [15] for the Rayleigh-Taylor instability of viscous fluid layers can be deduced from equation (3.6) by setting $H_0 = \eta_1 = \eta_2 = 0$ (η_1 and η_2 are the Darcy's coefficients of the two fluid layers), for two superposed magnetic fluids between two parallel plates [16] by setting $\eta_1 = \eta_2 = \mu_1 = \mu_2 = 0$ and for two semi- infinite superposed fluids through a porous medium [17] by setting $\mu_{1,2} = 0$, $H_0 = 0$ and $h_{1,2} \longrightarrow \infty$.

We consider now the general case of two superposed magnetic fluids, each of a finite thickness, in the presence of viscosity and Darcy's coefficients. The dispersion relation (3.6) is rewritten in the form, $D(\omega, k) = 0$,

$$a_0\omega^2 + ia_1\omega - a_2 = 0, \tag{3.7}$$

where

$a_0 = \rho_1 \coth kh_1 + \rho_2 \coth kh_2$,

$a_1 = \eta_1 \coth kh_1 + \eta_2 \coth kh_2 + 2k^2(\mu_1 \coth kh_1 + \mu_2 \coth kh_2)$,

$a_2 = k[g(\rho_1 - \rho_2) + kH_0^2(\tilde{\mu}_2 - \tilde{\mu}_1)^2(\tilde{\mu}_2 \coth kh_2 + \tilde{\mu}_1 \coth kh_1)^{-1} + \sigma k^2]$.

It is clear that the magnetic field has a stabilizing influence on the wave motion.This theoretical result was first obtained and confirmed experimentally by Zelazo and Melcher (see Rosensweig [1]). In addition, the viscous effects, as well as, the Darcy's coefficient influence have a destabilizing role [14, 15, 17].

Applying the Routh - Hurwitz criterion to (3.7) we obtain conditions for stability (in other words, having the imaginary part of ω to be less than zero):

$$a_1 > 0 \qquad and \qquad a_2 > 0, \tag{3.8}$$

since a_0 is always positive.

From above, we notice that the condition $a_1 > 0$ is trivially satisfied, while the condition $a_2 > 0$ reduces to

$$H_0^2 > [g(\rho_2 - \rho_1) - \sigma k^2](\tilde{\mu}_1 \coth kh_1 + \tilde{\mu}_2 \coth kh_2)/[k(\tilde{\mu}_1 - \tilde{\mu}_2)^2] \ . \tag{3.9}$$

For values of $H_0 > H_c$ (the critical magnetic field), where

$$H_c^2 = [g(\rho_2 - \rho_1) - \sigma k^2](\tilde{\mu}_1 \coth kh_1 + \tilde{\mu}_2 \coth kh_2)/[k(\tilde{\mu}_1 - \tilde{\mu}_2)^2] \ , \tag{3.10}$$

the system is linearly stable. But, for $H_0 < H_c$ the system is unstable. Thus, the critical magnetic field H_c is the linear cutoff magnetic field separating stable from unstable disturbances. In the following, we 'll study the growth in time of perturbations of the free surface throughout the fluid at the critical magnetic field.

4. THE NONLINEAR AMPLITUDE EQUATION

With view to deriving the equation for the evolution of the amplitude modulation of a travelling wave packet, we proceed to solve the second - and third - order problems. Following the procedure developed by Nayfeh [9], the nonsecularity conditions for the existence of uniformly valid solutions in the second-order problem are

$$-\frac{\partial D}{\partial \omega}\frac{\partial A}{\partial t_1} + \frac{\partial D}{\partial k}\frac{\partial A}{\partial x_1} = 0, \tag{4.1}$$

and its complex conjugate relation. If $\partial D/\partial \omega \neq 0$, the above equation becomes

$$\frac{\partial A}{\partial t_1} + \frac{d\omega}{dk}\frac{\partial A}{\partial x_1} = 0, \tag{4.2}$$

where $d\omega/dk = -(\partial D/\partial k)/(\partial D/\partial \omega)$ is the group velocity of the wave packet. It follows, as usual, that the amplitude A depends on the slow variables x_1 , t_1 through the combination $x_1 - (d\omega/dk)t_1$.

To develop the amplitude modulation for the progressive waves, we need to go to the third - order problem. By substituting the first- and second -

order solutions into the third - order one, we obtain the following solvability condition for the perturbation γ_3 to be nonsecular,

$$i(-\frac{\partial D}{\partial\omega}\frac{\partial A}{\partial t_2}+\frac{\partial D}{\partial k}\frac{\partial A}{\partial x_2})+\frac{1}{2}\frac{\partial^2 D}{\partial\omega^2}\frac{\partial^2 A}{\partial t_1^2}-\frac{\partial^2 D}{\partial\omega\partial k}\frac{\partial^2 A}{\partial x_1\partial t_1}+\frac{1}{2}\frac{\partial^2 D}{\partial k^2}\frac{\partial^2 A}{\partial x_1^2}=G|A|^2A, \tag{4.3}$$

where the coefficients of the linear terms are simply the derivatives of the characteristic function(3.6), while the coefficient of the nonlinear term is

$$\begin{aligned}G=&2\Lambda\{\omega^2[\rho_1(\coth^2kh_1+0.5csch^2kh_1)-\rho_2(\coth^2kh_2+0.5csch^2kh_2)]\\&+i\omega\Big(\eta_1\coth^2kh_1-\eta_2\coth^2kh_2+2k^2[\mu_1csch^2kh_1-\mu_2csch^2kh_2]\Big)\\&+k^2H_0^2\delta_1(k)\}+2k\{\omega^2[\rho_1\coth kh_1(1-csch^2kh_1)+\rho_2\coth kh_2(1-\\&csch^2kh_2)]+i\omega\Big(\eta_1\coth kh_1(1-0.5\coth^2kh_1)+\eta_2\coth kh_2(1-\\&0.5\coth^2kh_2)+k^2[\mu_1\coth kh_1(4-\coth^2kh_1)+\mu_2\coth kh_2(4-\\&\coth^2kh_2)]\Big)\}-2k^3H_0^2\delta_2(k)-1.5\sigma k^4,\end{aligned} \tag{4.4}$$

where

$$\begin{aligned}\Lambda=&\{\omega^2[\rho_2(\coth^2kh_2+0.5csch^2kh_2)-\rho_1(\coth^2kh_1+0.5csch^2kh_1)]\\&+0.5i\omega\Big(\eta_2csch^2kh_2-\eta_1csch^2kh_1+4k^2[\mu_2(\coth^2kh_2+csch^2kh_2)\\&-\mu_1(\coth^2kh_1+csch^2kh_1)]\Big)-k^2H_0^2\delta_1(k)\}/D(2\omega,2k),\end{aligned} \tag{4.5}$$

$$\begin{aligned}\delta_1(k)=&(\tilde\mu_2-\tilde\mu_1)\delta_0(k)[\frac{2}{\tilde\mu(2k)}+\frac{1}{2\tilde\mu(k)}-\frac{2\tilde\mu(k)}{(\tilde\mu_2-\tilde\mu_1)^2}+\frac{1}{2(\tilde\mu_2-\tilde\mu_1)\tilde\mu(k)}\\&(\tilde\mu_2\coth^2kh_2-\tilde\mu_1\coth^2kh_1)+\frac{2\tilde\mu_1\tilde\mu_2}{(\tilde\mu_2-\tilde\mu_1)^2\tilde\mu(2k)}(\coth kh_1+\coth kh_2)\\&\times(\coth 2kh_1+\coth 2kh_2)],\end{aligned} \tag{4.6}$$

$$\delta_2(k)=\delta_0(k)\{2-\frac{2(\tilde\mu_2-\tilde\mu_1)^2}{\tilde\mu(k)\tilde\mu(2k)}-\frac{(\tilde\mu_2-\tilde\mu_1)}{\tilde\mu(k)\tilde\mu(2k)}(\tilde\mu_1\coth kh_1\coth 2kh_1-\tilde\mu_2$$

$$\times \coth kh_2 \coth 2kh_2) + \frac{\tilde{\mu}_1\tilde{\mu}_2}{\tilde{\mu}(k)\tilde{\mu}(2k)}(\coth kh_1 + \coth kh_2)[(\coth 2kh_1$$
$$+ \coth 2kh_2) - 2\coth 2kh_1 \coth 2kh_2(\coth kh_1 + \coth kh_2)]\} . \tag{4.7}$$

Note that the asymptotic expansions break down when the denominator in (4.5) is zero, which corresponds to the second - harmonic resonance. Here we have assumed that $D(2\omega, 2k) \neq 0$.

The solvability conditions(4.1) and (4.3) can be simplified and combined together to produce a single equation. By using (4.2), derivatives in t_1 can be eliminated from equation (4.3). From (4.2), let 's write

$$\frac{\partial^2 A}{\partial x_1 \partial t_1} = -\frac{d\omega}{dk}\frac{\partial^2 A}{\partial x_1^2} \quad , \quad \frac{\partial^2 A}{\partial t_1^2} = (\frac{d\omega}{dk})^2 \frac{\partial^2 A}{\partial x_1^2} . \tag{4.8}$$

Substituting (4.8) into (4.3), dividing through by $(-\partial D/\partial\omega)$, and replacing x_m and t_m by $\epsilon^m x$ and $\epsilon^m t$, respectively, we have

$$i(\frac{\partial A}{\partial t} + \frac{d\omega}{dk}\frac{\partial A}{\partial x}) + P\frac{\partial^2 A}{\partial x^2} = \epsilon^2 Q|A|^2 A, \tag{4.9}$$

where the group velocity rate P and the nonlinear interaction coefficient Q are

$$P = \frac{1}{2}\frac{d^2\omega}{dk^2}$$
$$= -\frac{1}{2}[\frac{\partial^2 D}{\partial k^2}(\frac{\partial D}{\partial\omega})^2 - 2\frac{\partial D}{\partial k}\frac{\partial D}{\partial\omega}\frac{\partial^2 D}{\partial k\partial\omega} + \frac{\partial^2 D}{\partial\omega^2}(\frac{\partial D}{\partial k})^2]/(\frac{\partial D}{\partial\omega})^3,$$

and

$$Q = -G/(\frac{\partial D}{\partial\omega}) .$$

Introducing the transformation

$$X = \epsilon\,(x - \frac{d\omega}{dk}t) \quad , \quad T = \epsilon^2 t \ , \tag{4.10}$$

equation (4.9) is reduced to

$$i\frac{\partial A}{\partial T} + P\frac{\partial^2 A}{\partial X^2} = Q|A|^2 A \ , \tag{4.11}$$

which is a complex Ginzburg-Landau equation, i. e.

$$P = P_r + iP_i \qquad and \qquad Q = Q_r + iQ_i \ .$$

The complex Ginzburg-Landau equation (4.11) has a long history in physics, as a generic amplitude equation, near the onset of instabilities that lead to chaotic dynamics in numerous physical systems [18,19]. Analytical solutions are found by Landman [20]. He studied a particular class of solutions, which are called quasi-steady solutions, and found that their spatial variation may be periodic, quasi-periodic, or apparently chaotic. The stability of the complex Ginzburg-Landau equation(4.11) is discussed by Lange and Newell [21] and Matkowsky and Volpert [22]. They showed that stability conditions are

$$P_r Q_r + P_i Q_i > 0 \qquad and \qquad Q_i < 0 \ . \tag{4.12}$$

To study the stability of the system in the neighbourhood of the linear critical magnetic field(3.10), we notice that $P_r = Q_r = 0$ and therefore the complex Ginzburg- Landau equation(4.11) is reduced to the nonlinear diffusion equation,

$$\frac{\partial A}{\partial T} + P_i \frac{\partial^2 A}{\partial X^2} = Q_i |A|^2 A \ , \tag{4.13}$$

where the coefficients P_i and Q_i are the imaginary parts of P and Q, respectively, near the marginal state.

The solution of the nonlinear diffusion equation (4.13) is valid near the marginal state (i.e. $H_0 \longrightarrow H_c$)and can, therefore, be used to study the stability of the system. From the inequalities (4.12), we find the stability conditions of equation (4.13) as

$$P_i < 0 \qquad and \qquad Q_i < 0 \ . \tag{4.14}$$

Thus, if the above conditions (4.14) are satisfied, the finite deformation of the interface is stable and finite amplitude waves can be propagated through the interface. We 'll discuss the implications of conditions (4.14) in the following section.

5. STABILITY ANALYSIS

As shown above the analysis of the system for finite disturbance depends on (4.14). The stability can, therefore, be discussed by dividing the $H_0^2 - k$ plane into stable and unstable regions. The transition curves are given by the vanishing of P_i and Q_i. These curves are

$$A_1 H_0^4 + A_2 H_0^2 + A_3 = 0 \ , \tag{5.1}$$

and

$$\frac{A_4 H_0^4 + A_5 H_0^2 + A_6}{A_7 H_0^2 + A_8} = 0 \ , \tag{5.2}$$

where the A's are functions of $\rho_{1,2}, \tilde{\mu}_{1,2}, \eta_{1,2}, \mu_{1,2}, h_{1,2}, g$, σ and k. We observe that the condition (5.2) splits into

$$A_4 H_0^4 + A_5 H_0^2 + A_6 = 0 \ , \qquad \textit{and} \qquad A_7 H_0^2 + A_8 = 0 \ . \tag{5.3}$$

From equations (5.1), (5.3) and inequalities (4.14), we find that the system is stable provided that the magnetic field satisfies either the following conditions

$$A_1 H_0^4 + A_2 H_0^2 + A_3 < 0 \ , \quad A_4 H_0^4 + A_5 H_0^2 + A_6 < 0 \ , \quad A_7 H_0^2 + A_8 > 0 \ , \tag{5.4}$$

or

$$A_1 H_0^4 + A_2 H_0^2 + A_3 > 0 \ , \quad A_4 H_0^4 + A_5 H_0^2 + A_6 > 0 \ , \quad A_7 H_0^2 + A_8 < 0 \ . \tag{5.5}$$

The curve $A_7 H_0^2 + A_8 = 0$, or

$$H_0^2 = [g(\rho_2 - \rho_1) - 4\sigma k^2]/[2k\delta_0(2k)] \ , \tag{5.6}$$

is the second-harmonic internal resonance. Therefore, equation (5.6) represents a third transition curve in the stability charts. The phenomenon of internal resonance arises because of the occurrence of zero divisors in the representation of the second-order solutions. The analysis given, in this paper, is not valid in the neighbourhood of such a resonance.

The graphs represented by equations (5.1) and (5.2) are useful in studying the effects of the horizontal magnetic field, the viscosity and the Darcy's coefficients on the stability of the system, for various values of h_1 and h_2. Also the linear stability curve (dotted-line) representing relation (3.10) is given , which is assumed to divide the plane into an unstable region, symbolized by U (below the curve) and a stable region, symbolized by S (above the curve). The shaded regions are newly formed regions and are due to the nonlinear effect:S_1 and S_2 are stable and U_1, U_2 and U_3 are unstable regions. We observe that the second - harmonic resonance curve, (the broken-line) given by equation (5.6), is independent of both the viscosity and Darcy's coefficients.

Figures $1a$ – $1d$ are the stability diagrams corresponding to the cases $h_{1,2} \longrightarrow \infty$, $h_1 > h_2$, $h_1 = h_2$, and $h_1 < h_2$, respectively. For the case of two semi- infinite fluids, i.e. $h_1, h_2 \longrightarrow \infty$, we observe that the curve of the second - harmonic resonance lies below the linear curve. We, also, observe that the curve of the group velocity rate $P_i = 0$ has one branch, which cuts the linear curve, the resonance curve and the curve $Q_i = 0$. The latter curve lies below the resonance curve. Therefore, three new regions (U_1, U_2, S_1), due to nonlinear effects, have appeared. The first unstable region U_1 lies above the linear curve and below the curve $P_i = 0$. The second unstable region U_2 lies above both the linear curve and the curve $P_i = 0$, and the stable region S_1 lies among the curve $Q_i = 0$. the resonance curve, and the curve $P_i = 0$.

Figure $1b$ represents the same system considered in figure $1a$ but with $h_1 = 1.0cm$ and $h_2 = 0.1cm$ (i.e. $h_1 > h_2$). The behaviour of the system is similar to that in figure $1a$ with a decrease in the region U_1 and an increase in the region S_1. For $h_1 = h_2 = 0.1cm$ (figure $1c$) we see that the curve $Q_i = 0$ has two branches. One of them intersects with the linear curve and produces a new stable region S_2. The other branch lies below the resonance curve. This branch disappears in figure $1d$, where $h_1 < h_2$. We also notice that (in figure $1d$) the region U_2 disappears, while the new regionU_3 appears and lies below the curve$Q_i = 0$ and above the linear curve. In these figures, we see that the system is stabilizing for larger values of H_0^2 with smaller values of k, and destabilizing for larger values of k.

Figures $2a - 2d$ are the stability diagrams corresponding to the cases $h_{1,2} \longrightarrow \infty$, $h_1 > h_2$, $h_1 = h_2$, and $h_1 < h_2$, respectively. In these figures, we considered $\eta_1 = \eta_2 = 0$ (i.e. in the absence of Darcy's coefficients). For the case of two semi- infinite fluids, i.e. $h_1, h_2 \longrightarrow \infty$, we observe that the curve of the second - harmonic resonance lies below the linear curve. We, also, observe that the curve $Q_i = 0$ has one branch, which lies below the resonance curve, while the two branches of the group velocity rate $P_i = 0$ disappear. Therefore, two new regions (U_1, S_1), due to nonlinear effects, have appeared. The unstable region U_1 lies above the linear curve, while the stable region S_1 lies between the curve $Q_i = 0$ and the resonance curve.

Figure $2b$ represents the same system considered in figure $2a$ but with $h_1 = 1.0cm$ and $h_2 = 0.1cm$ (i.e. $h_1 > h_2$). The behaviour of the system is similar to that in figure $2a$ with a decrease in the region U_1 and an increase in the region S_1. For $h_1 = h_2 = 0.1cm$ (figure $2c$) we see that the curve $Q_i = 0$ has two branches. One of them intersects with the linear curve and produces a new stable region S_2. The other branch lies below the resonance curve. This branch disappears in figure $2d$, where $h_1 < h_2$. We also notice that the region U_1 disappears, while the new region U_2 appears and lies below the curve $Q_i = 0$ and above the linear curve.

6. NONLINEAR SCHRÖDINGER EQUATION

A special case occurs when the viscosity and *Darcy's* coefficients are negligible by taking $\mu_1 = \mu_2 = \eta_1 = \eta_2 = 0$ in the evolution equation (4.11). In this case, P_i and Q_i in equation (4.11) are equal to zero. Therefore, equation (4.11) is reduced to the nonlinear Schrödinger equation

$$i\frac{\partial A}{\partial T} + P_r\frac{\partial^2 A}{\partial X^2} = Q_r|A|^2A, \tag{6.1}$$

where

$$\begin{aligned}
P_r = {} & \frac{1}{2}\{\{\frac{2\omega}{k}(\rho_1\coth kh_1 + \rho_2\coth kh_2)\}^{-1}\{2\sigma + H_0^2\Big(k\delta_0''(k) + 2\delta_0'(k)\Big) - \frac{2\omega^2}{k^3} \\
& \times(\rho_1\coth kh_1 + \rho_2\coth kh_2) - \frac{2\omega^2}{k^2}(\rho_1 h_1 csch^2 kh_1 + \rho_2 h_2 csch^2 kh_2) - \frac{2\omega^2}{k} \\
& \times(\rho_1 h_1^2\coth kh_1 csch^2 kh_1 + \rho_2 h_2^2\coth kh_2 csch^2 kh_2)\} + 2\{\frac{2\omega}{k}(\rho_1\coth kh_1 \\
& + \rho_2\coth kh_2)\}^{-2}\{\frac{2\omega}{k^2}(\rho_1\coth kh_1 + \rho_2\coth kh_2) + \frac{2\omega}{k}(\rho_1 h_1 csch^2 kh_1 + \rho_2 \\
& \times h_2 csch^2 kh_2)\}\{2\sigma k + H_0^2\Big(k\delta_0'(k) + \delta_0(k)\Big) + \frac{\omega^2}{k^2}(\rho_1\coth kh_1 + \rho_2\coth kh_2) \\
& + \frac{\omega^2}{k}(\rho_1 h_1 csch^2 kh_1 + \rho_2 h_2 csch^2 kh_2)\} + \{\frac{2\omega}{k}(\rho_1\coth kh_1 + \rho_2\coth kh_2)\}^{-3} \\
& \times\{\frac{-2}{k}(\rho_1\coth kh_1 + \rho_2\coth kh_2)\}[2\sigma k + H_0^2\Big(k\delta_0'(k) + \delta_0(k)\Big) + \frac{\omega^2}{k^2}(\rho_1 \\
& \times\coth kh_1 + \rho_2\coth kh_2) + \frac{\omega^2}{k}(\rho_1 h_1 csch^2 kh_1 + \rho_2 h_2 csch^2 kh_2)]^2\},
\end{aligned}$$

$$\begin{aligned}
Q_r = {} & -\{\frac{2\omega}{k}(\rho_1\coth kh_1 + \rho_2\coth kh_2)\}^{-1}\{2[\omega^2\Big(\rho_2(\coth^2 kh_2 + 0.5csch^2 kh_2) \\
& - \rho_1(\coth^2 kh_1 + 0.5csch^2 kh_1)\Big) - k^2 H_0^2\delta_1(k)]^2[g(\rho_1 - \rho_2) + 2kH_0^2\delta_0(2k) \\
& + 4\sigma k^2 - \frac{2\omega^2}{k}(\rho_1\coth 2kh_1 + \rho_2\coth 2kh_2)]^{-1} + 1.5\sigma k^4 + 2k^3 H_0^2\delta_2(k) \\
& - 2k\omega^2[\rho_1\coth kh_1(1 - csch^2 kh_1) + \rho_2\coth kh_2(1 - csch^2 kh_2)]\},
\end{aligned}$$

and

$$\omega^2 = k[g(\rho_1 - \rho_2) + kH_0^2\delta_0(k) + \sigma k^2]/(\rho_1 \coth kh_1 + \rho_2 \coth kh_2).$$

Equation(6.1) describes the modulation of a one- dimensional weakly nonlinear dispersive wave in the presence of an externally applied magnetic field and in the absence of both the viscosity and Darcy's coefficients. It is well known that the solutions of this equation(6.1) are stable if and only if

$$P_r Q_r > 0 \,. \tag{6.2}$$

Thus, a finite- amplitude wave propagating through the surface is stable when the condition given by (6.2)is satisfied. This condition depends on k, g, σ. H_0. $h_{1,2}$, $\rho_{1,2}$ and $\tilde{\mu}_{1,2}$. The critical values of these parameters required for stability may be obtained from the equality of condition (6.2), namely

$$P_r Q_r = 0 \,. \tag{6.3}$$

The last condition (6.3) is given by the vanishing of both P_r and Q_r, where $P_r = 0$ and $Q_r = 0$ are polynomials of the second degree in H_0^2.

Figure3a shows the stability diagram for the two semi- infinite magnetic fluids case(i.e. $h_{1,2} \longrightarrow \infty$). In this case, the condition $P_r = 0$ gives

$$H_0^2 = [g^2(\rho_1 - \rho_2)^2 - 6\sigma g k^2(\rho_1 - \rho_2) - 3\sigma^2 k^4]/(4k^3\sigma\delta_0) \ , \tag{6.4}$$

where

$$\delta_0 = (\tilde{\mu}_2 - \tilde{\mu}_1)^2/(\tilde{\mu}_2 + \tilde{\mu}_1),$$

while the condition $Q_r = 0$ gives

$$2g(\rho_1 - \rho_2) + 0.5\sigma k^2 + 2(\rho_1 - \rho_2)^2\{g(\rho_1 - \rho_2) + \sigma k^2 + k\delta_0 H_0^2[1 + (\tilde{\mu}_2 - \tilde{\mu}_1)$$
$$\times(\rho_1+\rho_2)/(\tilde{\mu}_2+\tilde{\mu}_1)(\rho_1-\rho_2)]\}^2/\{(\rho_1+\rho_2)^2[g(\rho_1-\rho_2)-2\sigma k^2]\} = 0 \,. \tag{6.5}$$

We may observe that

$$k^2 = g(\rho_1 - \rho_2)/2\sigma \ , \tag{6.6}$$

is the second- harmonic resonance. Then the curve given by equation (6.6) is independent of the magnetic field. Also, this curve does not appear in this figure(3a) because we take $\rho_2 > \rho_1$. For this reason, the two roots of $Q_r = 0$ are complex numbers, and therefore, the two branches of $Q_r = 0$ does not appear in the graph, while the curve $P_r = 0$ has one branch. We also observe that the graph is divided into stable regions $(S, S_1)$and an unstable region U_1. The unstable region U_1 lies above the curve $P_r = 0$. The first stable region S_1 lies below the linear curve (the dotted-line), while the second region S lies between the linear curve and the curve $P_r = 0$. Therefore, the field is stabilizing for smaller values of k.

Figure 3b represents the same system considered in figure 3a, but with finite thickness, i.e. $h_1 = 1.0cm$ and $h_2 = 0.1cm$ (i.e. $h_1 > h_2$). We, also, observe that the curve of the second harmonic resonance does not appear. The branch of $Q_r = 0$ lies above the linear curve and cuts the upper branch of $P_r = 0$. Therefore, two new unstable regions U_1 and U_2 appear. The lower branch of $P_r = 0$ lies below the linear curve creating a new stable region S_1, which lies below the lower branch of the curve $P_r = 0$. The other stable region S_2 lies between the linear curve and the upper branch of the curve $P_r = 0$, for small values of k. The instability region has a larger range for larger values of the magnetic field.

Figure 3c represents the system considered in figure 3b but with $h_1 = 0.1cm$ (i.e. $h_1 = h_2$). We observe that there are two branches of the curve $P_r = 0$ and only one branch of the curve $Q_r = 0$. The curve $Q_r = 0$ lies above the linear curve and below the upper branch of the curve $P_r = 0$. The lower branch of $P_r = 0$ lies below the linear curve and makes one stable region S_1. This region lies below the curve $P_r = 0$. Comparing this graph with that of figure 3b, we see that the stable region S is increased while the unstable region U_2 is decreased. Also, the stable region S_2 disappears. Thus, the system has a stabilizing effect as the thickness of the lower fluid decreases.

Figure 3d represents the same system but with $h_1 = 0.01cm$. In this figure, we observe that the curve $Q_r = 0$ has one branch which lies above the linear curve and produces an unstable region U_1, while the curve $P_r = 0$ does not appear. The new unstable region lies above the curve $Q_r = 0$. This region increases with the increase of k while the new stable region S_1, which is lying below the linear curve, decreases as k increases.

From these graphs we see that the system is destabilizing for larger values of both H_0^2 and k, and stabilizing for smaller values of both H_0^2 and k.

7. CONCLUSION

The nonlinear instability of two finite- thickness superposed incompressible magnetic fluids in porous media under an applied horizontal magnetic field is investigated. The system is governed by the continuity equation, the Darcy's law and the Maxwell's equations (without free currents) in each fluid. Darcy's law (2.2) describes the weakly nonlinear evolution of the viscous fingering patterns obtained in a Hele- Shaw cell. This study, describes the Rayleigh- Taylor instability, with a low viscosity fluid pushing a more viscous one in a Hele- Shaw cell [27]. In a viscous potential flow (viscous potential flow [25] arises from the kinematic assumption that the curl of the fluid velocity vanishes identically in some region of space) the fluid velocity is given by the gradient of a velocity potential [27,28]. The effects of surface tension and weak viscosity are considered. The approximate picture is rigorously justified for the case of two fluids for which the difference in viscosity is very small. Therefore, the only place where the viscosity enters is in the normal component of the stress tensor. While the continuity of the tangential stress tensor across the interface is identically zero because of the continuity of the magnetic inductions and fields [29,30]. By using the method of multiple scaling we, obtain a dispersion relation in the linear approximation and a Ginzburg-Landau equation in the nonlinear approximation.

In the linear approximation, we have found that the magnetic field has a stabilizing effect, while both the viscosity and the Darcy's coefficients have a destabilizing effect.

In the nonlinear approximation, we have obtained the nonlinear diffusion equation when the linearized magnetic field is assumed to be nearly equal to the critical magnetic field. Further more, in section 6, it is shown that a nonlinear Schrödinger equation is obtained in the absence of both the viscosity and the Darcy's coefficients.

From the numerical discussion it is evident that, besides the effect of the variation of the thickness of the two magnetic fluids, the viscosity and the Darcy's coefficients play an important role in the nonlinear stability criterion of the problem.

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CAPTION OF FIGURES

Figure 1. Stability diagram in the $\log H_0^2 - k$ plane for a system having $\rho_1 = 0.0012 g/cm^3$, $\rho_2 = 0.9856 g/cm^3$, $\tilde{\mu}_1 = 1.007$, $\tilde{\mu}_2 = 5.0$, $\sigma = 0.06 dyn/cm$, $g = 981 cm/s^2$, $\mu_1 = 0.078$, $\mu_2 = 0.073$, $\eta_1 = 1.9812$, $\eta_2 = 0.914$, and $\omega = 0$, according to equation(4.15) .

(a) refers to $h_1, h_2 \longrightarrow \infty$, (b) to $h_1 = 1.0cm, h_2 = 0.1cm$,

(c) to $h_1 = h_2 = 0.1cm$, and (d) to $h_1 = 0.01cm, h_2 = 0.1cm$.

The dotted-line represents the linear curve while the broken-line represents the second-harmonic resonance curve. The symbols S and U denote stable and unstable regions, respectively, in the linear problem. Shaded regions are newly formed regions due to the nonlinear effects :S_j is stable and U_j is unstable regions.

Figure 2. Stability diagram for the same system considered in figure 1 but with $\eta_1 = 0$, $\eta_2 = 0$.

The symbols are the same as in figure 1.

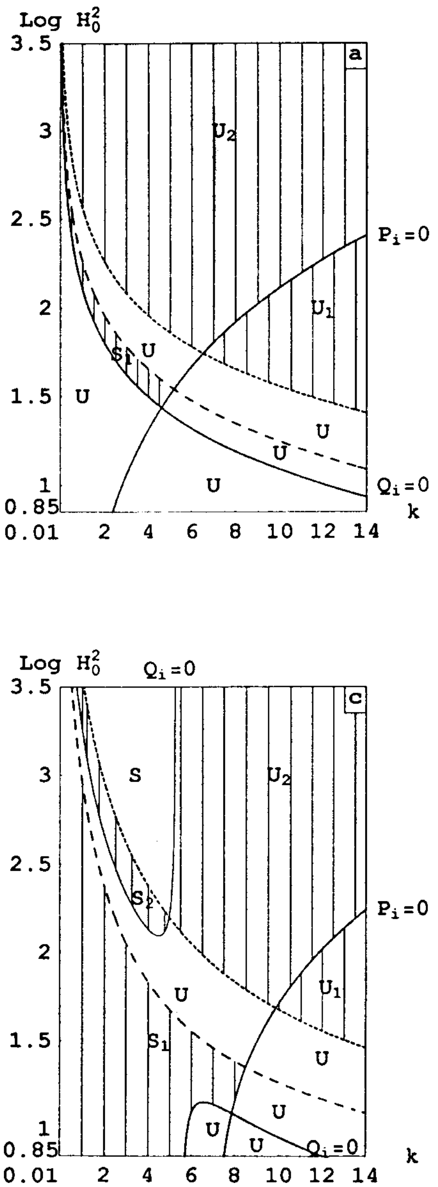
Figure 3. Stability diagram in the $\log H_0^2 - k$ plane for a system having $\rho_1 = 0.00036522 g/cm^3$, $\rho_2 = 0.596 g/cm^3$, $\tilde{\mu}_1 = 1.007$, $\tilde{\mu}_2 = 1.7$, $\sigma = 0.06 dyn/cm$ and $g = 981 cm/s^2$, according to equation(6.1).

(a) refers to $h_1, h_2 \longrightarrow \infty$, (b) to $h_1 = 1.0cm, h_2 = 0.1cm$,

(c) to $h_1 = h_2 = 0.1cm$, and (d) to $h_1 = 0.01cm, h_2 = 0.1cm$.

The symbols are the same as in figure 1.

Figure 1


Log H_0^2
a
U_2
U_1
S_1
U
$P_i=0$
$Q_i=0$
k
b
c
S_2
S
d
U_3
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3
2.5
2
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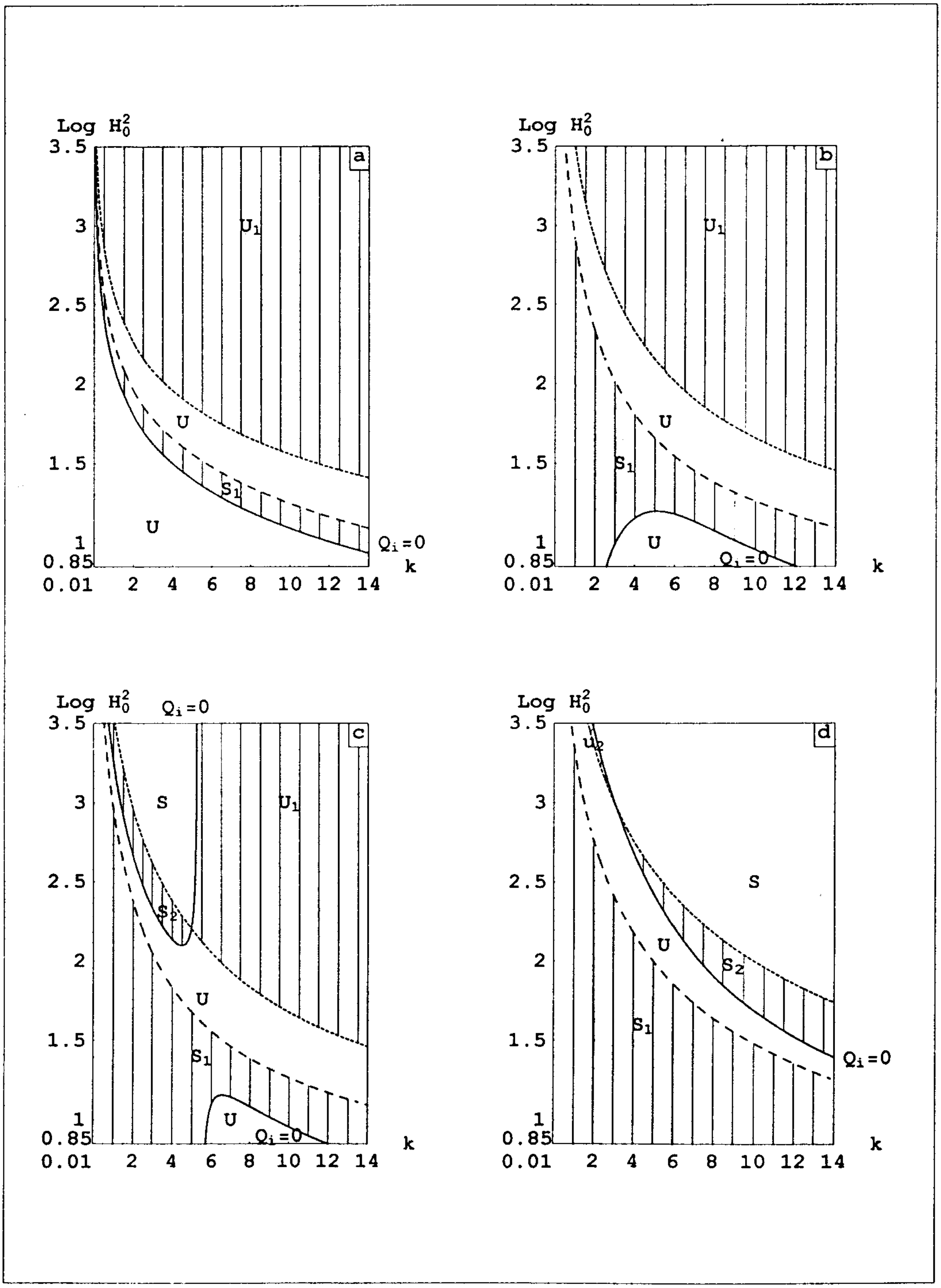
1
0.85
0.01 2 4 6 8 10 12 14

Figure 2


Log H_0^2
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U_1
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U
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k
b
U_1
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S_1
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$Q_i=0$
c
$Q_i=0$
S
U_1
S_2
U
S_1
U
$Q_i=0$
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u_2
S
U
S_2
S_1
$Q_i=0$
3.5
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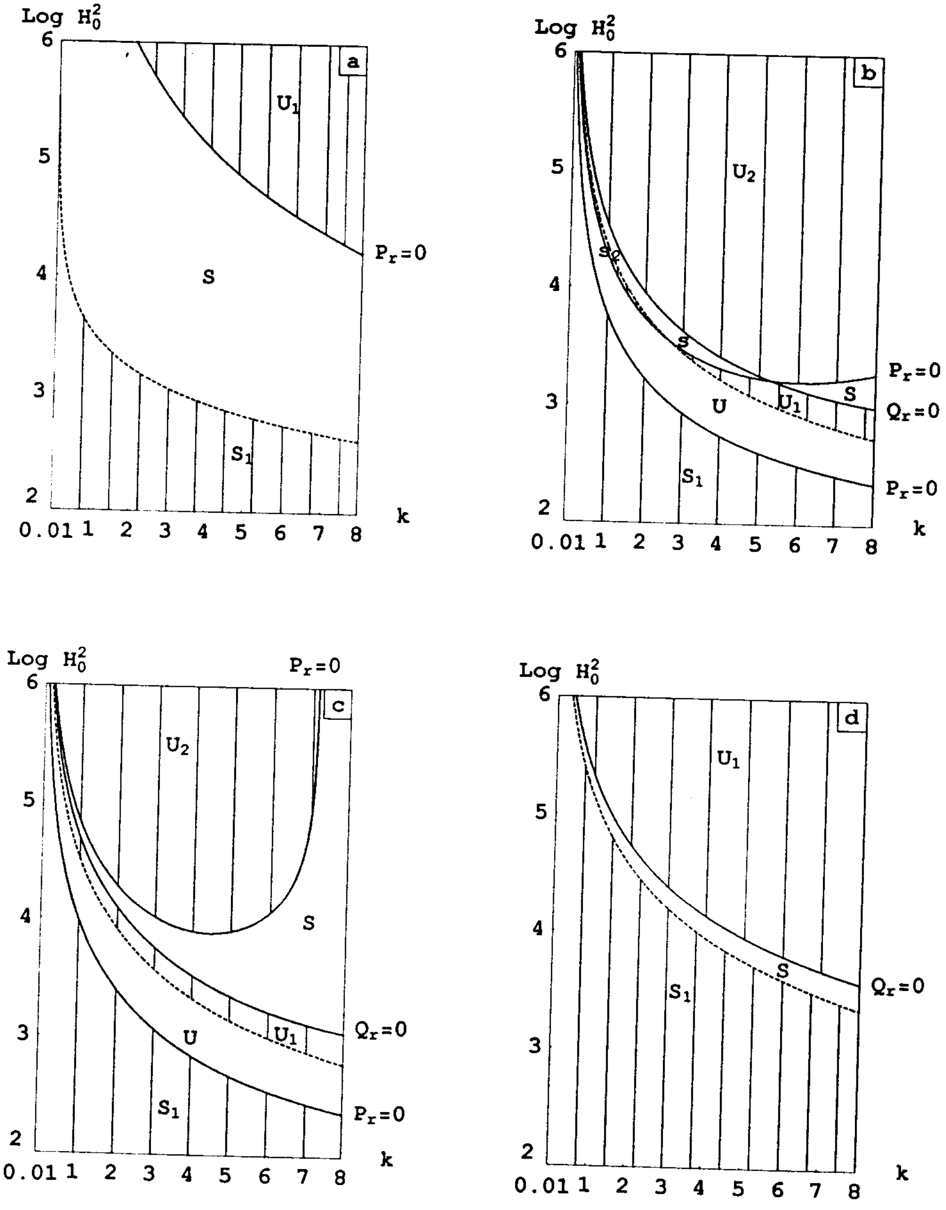
14

Figure 3


Log H_0^2
a
U_1
S
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$P_r=0$
k
0.01 1 2 3 4 5 6 7 8
2 3 4 5 6
b
U_2
U
U_1
S_1
$P_r=0$
$Q_r=0$
c
U_2
S
U
U_1
S_1
$Q_r=0$
d
U_1
S_1

عدم الإستقرار غير الخطي لموانع قوية التمغنط في أوساط مسامية في وجود مجال مغناطيسى أفقي

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يهدف هذا البحث إلى دراسةعدم الإستقرار غير الخطى للسطح الفاصل بين مائعين مغناطيسيين غير قابلين للانضغاط وضعيفي اللزوجة في وجود مجال مغناطيسي مماسي منتظم في أوساط مسامية ، وكل من المائعين ذو سمك محدد ، مع الأخذ في الأعتبار أن الحركة غير دورانية وتأثير اللزوجة الضعيف في الشروط الحدية • وقد أخذت تأثيرات كل من التوتر السطحي والجاذبية في الإعتبار • وبإستخدام طريقة التدرج المتضاعف تم دراسة استقرار النظام في الحالتين الخطية وغير الخطية . ففي الحالة الخطية تم الحصول على علاقة التشتت للنظام ومن ثم شروط الإستقرار ومنها تبين التأثير الإستقرارى للمجال المغناطيسي المماسي ، بينما معاملى اللزوجة والمسامية لهما تأثير غير إستقرارى . أما في الحالة غير الخطية فلقد تم الحصول على معادلة جينزبرج- لانداو والتي أستخدمت لوصف سلوك النظام من خلال شروط الإستقرار غير الخطي . وفي الحالة الخاصة عند إهمال تأثيرات اللزوجة والمسامية فإن معادلة جينزبرج – لانداو تؤول إلى معادلة شرودنجر غير الخطية .

ومن خلال المناقشات العددية في الحالة غير الخطية تبين أن كلا من اللزوجة ومسامية الوسط وكذا المجال المغناطيسي لهم دور مزدوج في التأثير على إستقرار النظام.