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ANOTHER DECOMPOSITIONS OF CONTINUITY AND SOME WEAKER FORMS OF CONTINUITY VIA IDEALIZATION

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ABSTRACT

The purpose of this paper is to give decompositions of continuity and some weaker forms of continuity via idealization using the concepts of αB_I -sets, αWLC_I -sets, AB_I -sets and WAB_I -sets.

1 INTRODUCTION AND PRELIMINARIES

In 1992, Jankovi´c and Hamlett [13] introduced the notion of I-open sets in ideal topological spaces. Abd El-Monsef *et al.* [2] further investigated I-open sets and I-continuous functions. In 1999, Dontchev [7] introduced the notion of pre-I-open sets which is weaker than that of I-open sets. Recently, Hatir and Noiri [8] have introduced the notions of B_I -sets, C_I -sets, α-I-open sets, semi-I-open sets and β -I-open sets. By using these sets, they provided decompositions of continuity. In this paper, we introduce the notions of αB_I - sets, αWLC_I -sets, AB_I -sets and WAB_I -sets to obtain decompositions of continuity and some weaker forms of continuity.

Throughout this paper, for a subset A of a space (X, τ), the closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. An ideal topological space is a topological space (X, τ) with an ideal I on X, and is denoted by (X, τ, I). The following collections form important ideals on a topological space (X, τ): The ideal of all finite sets *F*, the ideal of all closed and discrete sets *CD*, the ideal of all nowhere dense sets *N*. $A^*(I) = \{x \in X \mid U \cap A \notin I$ *for each open neighborhood U of x*$\}$ is called the local function of A with respect to I and τ [12]. When there is no chance for confusion $A^*(I)$ is denoted by A^*. Note that often X^* is a proper subset of X. The hypothesis $X = X^*$ was used by Hayashi [11], while the hypothesis $\tau \cap I = \varnothing$ was used by Samuels [18]. In fact, these two conditions are equivalent by Theorem 6.1 of [12] and so the ideal topological spaces satisfying this hypothesis are called as Hayashi-Samuels spaces.

_Key words : _-I-open, strong _-I-sets, ABI -sets, WABI -sets and _WLCI -sets
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For every ideal topological space (X, τ, I), there exits a topology $\tau^*(I)$, finer than τ, generated by the base $\beta(I, \tau) = \{U \setminus I \mid U \in \tau \text{ and } I \in I\}$. In general $\beta(I, \tau)$ is not always a topology [11]. Observe additionally that $Cl^*(A) = A^* \cup A$ defines a Kuratowski closure operator for $\tau^*(I)$. Now we recall some definitions and results, which are used in this paper.

Definition 1.1 *A subset* A of a topological space (X, τ) *is called*:

(a) an α*-open set* [17] *if* $A \subset Int(Cl(Int(A)))$,
(b) a semi-open set [14] *if* $A \subset Cl(Int(A))$,
(c) a pre-open set [15] *if* $A \subset Int(Cl(A))$,
(d) a β*-open set* [1] *if* $A \subset Cl(Int(Cl(A)))$,
(e) an αB*-set* [16] *if* $A = U \cap V$, *where* U *is* α*-open and* $Int(Cl(V)) = Int(V)$,
(f) an AB-set [6] *if* $A = U \cap V$, *where* U *is open and* $Int(Cl(V)) \subset V \subset Cl(Int(V))$,
(g) a WAB-set [9] *if* $A = U \cap V$, *where* U *is open and* $Int(Cl(Int(V))) \subset V \subset Cl(Int(CI(V)))$,
(h) a LC-set [3] *(resp.* αLC*-set* [16]*) if* $A = U \cap V$, *where* U *is open(resp.* α*-open) and* V *is closed,*
(i) a $D(p, ps)$*-set* [5] *if* $A \cap Cl(Int(Cl(A))) = A \cap Int(Cl(A))$.

Definition 1.2 *A subset A of an ideal topological space* (X, τ, I) *is said to be:*

(a) $*$*-perfect* [11] *if* $A = A^*$,
(b) α*-I-open* [8] *if* $A \subset Int(Cl^*(Int(A)))$,
(c) semi-I-open [8] *if* $A \subset Cl^*(Int(A))$,
(d) pre-I-open [7] *if* $A \subset Int(Cl^*(A))$,
(e) strongly β*-I-open* [10] *if* $A \subset Cl^*(Int(Cl^*(A)))$,
(f) an A_I*-set* [4] *if* $A = U \cap V$, *where* U *is open and* $Cl^*(Int(V)) = V$,
(g) a B_I*-set (resp. an* αB_I*-set)* [8] *if* $A = U \cap V$, *where* U *is open (resp.* α*-I-open) and* $Int(Cl^*(V)) = Int(V)$,
(h) a C_I*-set* [8] *if* $A = U \cap V$, *where* U *is open and* $Int(Cl^*(Int(V))) = Int(V)$,
(i) a WLC_I*-set (resp.* αWLC_I*-set)* [4] *if* $A = U \cap V$, *where* U *is open (resp.* α*-I-open) and* $Cl^*(V) = V$.

2 Decomposition of some weaker forms of continuity

Definition 2.1 *A subset A of an ideal topological space (X, τ, I) is called:*

(a) an αB_I-set if $A = U \cap V$, where U is α-I-open and $Int(Cl^*(V)) = Int(V)$,
(b) a $D(p,ps)_I$-set if $A \cap Int(Cl^*(A)) = A \cap Cl(Int(Cl^*(A)))$.

Every B_I-set is an αB_I-set but not conversely as shown by the following example.

Example 2.1 *Let $X = \{a,b,c\}$, $\tau = \{X, \varnothing, \{a\}\}$ and $I = \{\varnothing, \{c\}\}$. Then $A = \{a,c\}$ is an αB_I-set but it is not a B_I-set. For, $Int(Cl^*(Int(A)))$ $= Int(Cl^*(Int(\{a,c\}))) = Int(Cl^*(\{a\})) = Int(\{a\} \cup \{a\}^*) = Int(X) = X \supset$ A and A is α-I-open. Therefore A is an αB_I-set. On the other hand, $A \notin \tau$ and $Int(Cl^*(A)) = Int(Cl^*(\{a,c\})) = Int(X) = X \neq \{a\} = Int(A)$. Hence A is not a B_I-set.*

Proposition 2.1 *For any subset A of an ideal topological space X, the fol-lowingconditions are equivalent:*
(a) A is semi-I-open;
(b) There exits an open set U in (X, τ, I) such that $U \subset A \subset Cl^*(U)$.

Proof.

(a) ⇒ (b) Let A be a semi-I-open set. Then $A \subset Cl^*(Int(A))$ and put U = Int(A). Then U is open and $U \subset A \subset Cl^*(U)$.
(b) ⇒ (a) Let $U \subset A \subset Cl^*(U)$ for an open set U. This implies $Cl^*(Int(A))$ $= Cl^*(U)$. So $A \subset Cl^*(Int(A))$.

Proposition 2.2 *Let (X, τ, I) be an ideal topological space and let $A \subset (X, \tau, I)$. Then, A is semi-I-open if and only if $A = U \cap V$, where $U = Cl^*(Int(U))$ and $Cl^*(Int(V)) = X$.*

Proof.
Necessity. Let A be semi-I-open. By Proposition 2.1, $U \subset A \subset Cl^*(U)$ for some $U \in \tau$. Note that $Cl^*(U) = Cl^*(A)$. We write $A = Cl^*(U) \setminus (Cl^*(U) \setminus$

$A)) = Cl^*(U) \cap (X \setminus (Cl^*(U) \setminus A))$. Then, U = Int(U) $\subset Int(Cl^*(U)) \subset Cl^*(U)$. Therefore, $Cl^*(U) = Cl^*(Int(Cl^*(U)))$. Besides, $Cl^*(Int(X \setminus (Cl^*(U) \setminus A)))$ $= Cl^*(X \setminus Cl(Cl^*(U) \setminus A)) \supset Cl^*(X \setminus Cl(Cl^*(U) \setminus U)) = Cl^*(X \setminus (Cl^*(U) \setminus U))$ $= Cl^*((X \setminus Cl^*(U)) \cup U) = Cl^*(X \setminus Cl^*(U)) \cup Cl^*(U) \supset (X \setminus Cl^*(U)) \cup Cl^*(U)$ = X. So, $Cl^*(Int(X \setminus (Cl^*(U) \setminus A))) =$ X. Then, A = U $\cap$ V where U = $Cl^*(Int(U))$ and $Cl^*(Int(V)) =$ X.

Sufficiency. Assume that A = U $\cap$ V where U = $Cl^*(Int(U))$ and $Cl^*(Int(V))$ = X. We choose G $\in \tau$ such that U = $Cl^*(G)$. We put H = G $\cap$ Int(V). Then H $\in \tau$ with H $\subset$ A. Finally, $Cl^*(H) = Cl^*(G \cap Int(V)) = Cl^*(G)$. Therefore H $\subseteq$ A $\subset Cl^*(H)$ and A is semi-I-open.

Theorem 2.1 A subset A is semi-I-open in an ideal topological space *if and only if it is strong β-I-open and an αWLC_I-set.*

Proof.

Necessity. Let A be semi-I-open. By Proposition of [8], A is strong-β-I-open. By Proposition 2.2,. A = U $\cap$ V where U = $Cl^*(Int(U))$ and $Cl^*(Int(V))$ = X. Since $Int(Cl^*(Int(V))) =$ X $\supset$ V, V is α-I-open. Besides, $Cl^*(U) = Cl^*(Int(U)) =$ U. So A is an αWLC_I-set.

Sufficiency. Let A be strong β-I-open and an αWLC_I-set. Then A= U $\cap$ V, where U is α-I-open and $Cl^*(V)=$ V. By the definition of strong β-I-openness, we have A $\subset Cl^*(Int(Cl^*(A)))$. Then A $\subset$ U $\cap$ $Cl^*(Int(Cl^*(A)))$ $\subset$ U $\cap$ $Cl^*(Int(Cl^*(V)))$ = U $\cap$ $Cl^*(Int(V))$ $\subset$ U $\cap$ $Cl^*(V)$ = U $\cap$ V = A. Thus A = U $\cap$ $Cl^*(Int(Cl^*(A)))$, where U is α-I-open and by Proposition 2.1, $Cl^*(Int(Cl^*(A)))$ is semi-I-open. Therefore by Proposition 2.3 of [4], A is semi-I-open.

Theorem 2.2 *A subset A is α-I-open in an ideal topological space (X, τ, I), if and only if it is pre-I-open and an αB_I -set.*

Proof.

Necessity. It is obvious.

Suffciency. Let A be pre-I-open and an αB_I-set. Then A = U $\cap$ V, where U is α-I-open and $Int(Cl^*(V)) = Int(V)$. By the definition of pre-I-openness, we have A $\subset Int(Cl^*(A))$. Then, A $\subset$ U $\cap$ $Int(Cl^*(U \cap V))$ $\subset$ U $\cap$ $Int(Cl^*(U))$ $\cap$ $Int(Cl^*(V))$ = U $\cap$ Int(V) $\subset$ A. Thus, A = U $\cap$ Int(V), where U is α-I-open and Int(V) is open. We obtain A is α-I-open.

Theorem 2.3 *A* subset *A* is pre-I-open in an ideal topological space (X,τ,I) *if and only if it is : β-I-open and a $D(p,ps)_I$-set*

Proof.

Necessity. It is obvious.

Suffciency. Let A be β-I-open and a $D(p,ps)_I$-set. By the definition of β-I-open, we have A $\subset Cl(Int(Cl^*(A)))$. Then A = A $\cap Cl(Int(Cl^*(A)))$ = A $\cap Int(Cl^*(A)) \subset Int(Cl^*(A))$. Thus A is pre-I-open.

Definition 2.2 *A function f: $(X,\tau,I) \to (Y,\sigma)$ is αB_I-continuous (resp. αWLC_I-continuous,D(p, ps)-I-continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is an αB_I-set (resp. an αWLC_I-set, a $D(p,ps)_I$-set) of (X,τ,I).*

Theorem 2.4 *Let (X,τ,I) be an ideal topological space. For a function f: $(X,\tau,I) \to (Y,\sigma)$, the following conditions are equivalent:*

(a) f is semi-I-continuous;

(b) f is strong β-I-continuous and αWLC_I-continuous.

Proof.

This is an immediate consequence of Theorem 2.1.

Corollary 2.1 ([16], Corollary 3.8) *Let* (X,τ,I) be an ideal topological space and $I = \{\varnothing\}$. *For a function f: $(X,\tau) \to (Y,\sigma)$, the following conditions* are equivalent:

(a) f is semi-continuous;

(b) f is β-continuous and αLC-continuous.

Proof.

Since $\mathbf{I} = \{\varnothing\}$, we have $A^* = \mathrm{Cl}(A)$ and $Cl^*(A) = A \cup A^* = \mathrm{Cl}(A)$ for any subset A of X. Therefore A is semi-I-open (resp. strong β-I-open, an αWLC_I-set) if and only if A is semi-open (resp. β-open, an αLC-set). The proof follows from Theorem 2.4 immediately.

Theorem 2.5 *Let* (X,τ,I) be an ideal topological space. *For a function f: $(X,\tau,I) \to (Y,\sigma)$, the following conditions are equivalent:*

(a) f is α-I-continuous;

(b) f is pre-I-continuous and αB_I-continuous.

Proof.

This is an immediate consequence of Theorem 2.2.

Corollary 2.2 ([16], Corollary 2.7) *Let (X, τ, I) be an ideal topological space and $I = \{\varnothing\}$. For a function f: $(X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*
(a) f is α-continuous;
(b) f is pre-continuous and αB-continuous.

Proof.

Since I = $\{\varnothing\}$, we have $A^* = \text{Cl(A)}$ and $Cl^*(A) = A \cup A^* = \text{Cl(A)}$ for any subset A of X. Therefore A is α-I-open (resp. pre-I-open, an αB_I-set) if and only if A is α-open (resp. preopen, an αB-set). The proof follows from Theorem 2.5 immediately.

Theorem 2.6 *Let (X, τ, I) be an ideal topological space. For a function f: $(X, \tau, I) \to (Y, \sigma)$, the following conditions are equivalent:*
(a) f is pre-I-continuous;
(b) f is β-I-continuous and D(p, ps)-I-continuous.

Proof.
This is an immediate consequence of Theorem 2.3.

Corollary 2.3 ([5]) *Let (X, τ, I) be an ideal topological space and $I = \{\varnothing\}$. For a function f: $(X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*
(a) f is pre-continuous;
(b) f is β-continuous and D(p,ps)-continuous.

Proof.

Since I = $\{\varnothing\}$, we have $A^* = \text{Cl(A)}$ and $Cl^*(A) = A \cup A^* = \text{Cl(A)}$ for any subset A of X. Therefore A is pre-I-open (resp. β-I-open, a $D(p, ps)_I$-set) if and only if A is pre-open (resp. β-open, a D(p,ps)-set). The proof follows from Theorem 2.6 immediately.

3 Decomposition of continuity

Definition 3.1 *A subset A of an ideal topological space (X, τ, I) is called an AB_I-set if $A = U \cap V$, where U is open and $Int(Cl^*(V)) \subset V \subset Cl^*(Int(V))$.*

Proposition 3.1 *Let (X, τ, I) be an ideal topological space. For a subset $A \subseteq X$, the following implications hold:*

$$A_I\text{-set} \Rightarrow AB_I\text{-set} \Rightarrow B_I\text{-set}.$$

Proof.

The proof is obvious.

None of these implications is reversible as the following examples show:

Example 3.1 *Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{b\}, \{d\}, \{b, d\}\}$ and $I = \{\varnothing, \{c\}\}$. Then $A = \{b, c\}$ is an AB_I-set, but it is not an A_I-set. Because, $Int(Cl^*(A)) = Int(Cl^*(\{b, c\})) = Int(\{b, c\} \cup \{b, c\}^*) = Int(\{a, b, c\}) = \{b\} \subset A$. Besides, $Cl^*(Int(A)) = Cl^*(Int(\{b, c\})) = Cl^*(\{b\}) = \{b\} \cup \{b\}^* = \{a, b, c\} \supset A$ and A is an AB_I-set. On the other hand, $A = \{b, c\} \notin \tau$ and $Cl^*(Int(A)) = Cl^*(Int(\{b, c\})) = \{a, b, c\} \neq A$. Hence A is not an A_I-set.*

Example 3.2 *Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{c, d\}, \{a, c, d\}\}$ and $I = \{\varnothing, \{a\}\}$. Then $A = \{a, b\}$ is a B_I-set, but it is not an AB_I-set. For, $Int(Cl^*(A)) = Int(Cl^*(\{a, b\})) = Int(\{a, b\} \cup \{a, b\}^*) = Int(\{a, b\}) = \{a\} = Int(A)$. So A is a B_I-set. On the other hand, $A = Cl^*(Int(A)) \notin \tau$ and $Cl^*(Int(A)) = Cl^*(Int(\{a, b\}) = Cl^*(\{a\}) = \{a\} \cup \{a\}^* = \{a\} \not\supset A$. So, A is not an AB_I-set.*

Theorem 3.1 *For a subset A of an ideal topological space (X, τ, I), the fol-lowing are equivalent:*

(a) A is an AB_I-set;

(b) A is semi-I-open and a B_I-set;

(c) A is strong β-I-open and a B_I-set.

Proof.

(a) $\Rightarrow$ (b) and (b) $\Rightarrow$ (c) are obvious.

(c) $\Rightarrow$ (a) Let A be strong β-I-open and a B_I-set. Since A is a B_I-set, A = U $\cap V_0$, where U is open and $Int(Cl^*(V_0)) = Int(V_0)$. Then A $\subset$ U $\cap$ (A $\cup$ $Int(Cl^*(A))) \subset$ U $\cap$ ($V_0 \cup Int(Cl^*(V_0))$) = U $\cap V_0$ = A. Hence A = U $\cap$ (A $\cup$ $Int(Cl^*(A))$). Besides, $Cl^*(Int(A \cup Int(Cl^*(A)))) \supset Cl^*(\text{Int}(A) \cup Int(Cl^*(A))) = Cl^*(Int(Cl^*(A)))$. Since A is strong β-I-open, $Cl^*(\text{Int}(A \cup Int(Cl^*(A)))) \supset A \cup Int(Cl^*(A))$. Furthermore, $Int(Cl^*(A \cup Int(Cl^*(A)))) = \text{Int}(Cl^*(A) \cup Cl^*(Int(Cl^*(A)))) \subset Int(Cl^*(A))$. Therefore $Int(Cl^*(A \cup Int(Cl^*(A)))) \subset A \cup Int(Cl^*(A)$. Put V = A $\cup Int(Cl^*(A))$. Then, A = U $\cap$ V, where U $\in \tau$ and $Int(Cl^*(V)) \subset V \subset Cl^*(Int(V))$. Hence A is an AB_I-set.

Theorem 3.2 *For a subset A of an ideal topological space* (X, τ, I), *the following* are equivalent:

(a) *A is open;*
(b) *A is pre-I-open and an* AB_I*-set;*
(c) *A is pre-I-open and a* B_I*-set.*

Proof.
(a) $\Rightarrow$ (b) and (b)$\Rightarrow$(c) are obvious by Proposition 3.1.
(c)$\Rightarrow$ (a) This follows from Proposition 3.3 in [8].

Definition 3.2 *A subset A of an ideal topological space* (X, τ, I) *is called a* WAB_I*-set if* $A = U \cap V$, *where U is open and* $Int(Cl^*(Int(V))) \subset V \subset Cl^*(Int(Cl^*(V)))$.

Every AB_I-set is a WAB_I-set but not conversely as shown by the following example.

Example 3.3 *Let* $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{b\}, \{a, d\}, \{a, b, d\}\}$ *and* $I = \{\varnothing, \{c\}\}$. *Then* $A = \{a, c\}$ *is a* WAB_I*-set, but it is not an* AB_I*-set.* $Int(Cl^*(Int(A))) = Int(Cl^*(Int(\{a, c\}))) = Int(Cl^*(\emptyset)) = \emptyset \subset A$. *Besides,* $Cl^*(Int(Cl^*(A))) = Cl^*(Int(Cl^*(\{a, c\}))) = Cl^*(Int(\{a, c\} \cup \{a, c\}^*)) = Cl^*(Int(\{a, c, d\})) = Cl^*(\{a, d\}) = \{a, d\} \cup \{a, d\}^* = \{a, c, d\} \supset A$. *Therefore, A is a* WAB_I*-set. On the other hand,* $A = \{a, c\} \notin \tau$ *and* $Cl^*(Int(A)) = Cl^*(Int(\{a, c\})) = Cl^*(\emptyset) = \emptyset \not\supset A$. *So, A is not an* AB_I*-set.*

Every WAB_I-set is strongly β-I-open and a C_I-set but not conversely as shown by the following examples.

Example 3.4 *Let* $X = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$ *and* $I = \{\varnothing, \{b\}\}$. *Then* $A = \{b, c\}$ *is a* C_I*-set, but it is not a* WAB_I*-set. For,* $Int(Cl^*(Int(A))) = Int(Cl^*(Int(\{b, c\}))) = Int(Cl^*(\{b\})) = Int(\{b\} \cup \{b\}^*) = Int(\{b\}) = \{b\} = Int(A)$. *So, A is a* C_I*-set. On the other hand,* $A = \{b, c\} \notin \tau$ *and* $Cl^*(Int(Cl^*(A))) = Cl^*(Int(Cl^*(\{b, c\}))) = Cl^*(Int(\{b, c\} \cup \{b, c\}^*)) = Cl^*(Int(\{b, c\})) = Cl^*(\{b\}) = \{b\} \cup \{b\}^* = \{b\} \not\supset A$. *Therefore, A is not a* WAB_I*-set.*

Example 3.5 *Let* $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{a, c\}\}$ *and* $I = \{\varnothing, \{b\}\}$. *Then* $A = \{a, c, d\}$ *is strong* β*-I-open, but it is not a* WAB_I*-set. Because,* $Cl^*(Int(Cl^*(A))) = Cl^*(Int(Cl^*(\{a, c, d\}))) = Cl^*(Int(\{a, c, d\} \cup \{a, c, d\}^*)) = Cl^*(Int(X)) = X \supset A$. *Therefore, A is strong* β*-I-open. On the other hand,*

$A = \{a, c, d\} \notin \tau$. *Besides,* $Int(Cl^*(Int(A))) = Int(Cl^*(Int(\{a, c, d\}))) = Int(Cl^*(\{a, c\})) = Int(\{a, c\} \cup \{a, c\}^*) = Int(X) = X \not\subset A$. *So, A is not a* WAB_I*-set.*

Theorem 3.3 *For a subset A of an ideal topological space (X, τ, I), the following are equivalent:*

(a) A is open;
(b) A is α-I-open and a WAB_I*-set;*
(c) A is α-I-open and a C_I*-set.*

Proof.
(a) ⇒ (b) and (b) ⇒ (c) are obvious.
(c)⇒ (a) This follows from Proposition 3.3 in [8].

Definition 3.3 *A function f:* $(X, \tau, I) \to (Y, \sigma)$ *is* AB_I*-continuous (resp.* WAB_I*-continuous) if for every* $V \in \sigma$, $f^{-1}(V)$ *is an* AB_I*-set (resp. a* WAB_I*-set) of* (X, τ, I).

Theorem 3.4 *Let* (X, τ, I) *be an ideal topological space. For a function f:* $(X, \tau, I) \to (Y, \sigma)$*, the following conditions are equivalent:*
(a) f is continuous;
(b) f is pre-I-continuous and AB_I*-continuous.*

Proof.
This is an immediate consequence of Theorem 3.2.

Corollary 3.1 ([5], Theorem 4.4) *Let* (X, τ, I) *be an ideal topological space and* $I = \{\varnothing\}$*. For a function f:* $(X, \tau) \to (Y, \sigma)$*, the following conditions are equivalent:*
(a) f is continuous;
(b) f is pre-continuous and AB-continuous.

Proof.
Since $I = \{\varnothing\}$, we have $A^* = Cl(A)$ and $Cl^*(A) = A \cup A^* = Cl(A)$ for any subset A of X. Therefore A is pre-I-open (resp. an AB_I-set) if and only if A is pre-open (resp. an AB-set). The proof follows from Theorem 3.4 immediately.

Theorem 3.5 *Let (X, τ, I) be an ideal topological space. For a function f: $(X, \tau, I) \to (Y, \sigma)$, the following conditions are equivalent:*
(a) f is continuous;
(b) f is α-I-continuous and WAB_I-continuous.

Proof.
This is an immediate consequence of Theorem 3.3.

Corollary 3.2 ([9], Theorem 4.1) *Let (X, τ, I) be an ideal topological space and $I = \{\varnothing\}$. For a function f: $(X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*
(a) f is continuous;
(b) f is α-continuous and WAB-continuous.

Proof.
Since $I = \{\varnothing\}$, we have $A^* = \mathrm{Cl}(A)$ and $Cl^*(A) = A \cup A^* = \mathrm{Cl}(A)$ for any subset A of X. Therefore A is α-I-open (resp. an WAB_I-set) if and only if A is α-open (resp. an WAB-set). The proof follows from Theorem 3.5 immediately.

Conclusions 3.1 *Let (X, τ, I) be an ideal topological space. For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*
(a) f is continuous,
(b) f is pre-I-continuous and AB_I-continuous,
(c) f is α-I-continuous and WAB_I-continuous.

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