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THE NEW BETTER THAN USED IN THE EXCESS WEALTH ORDER CLASS OF LIFE DISTRIBUTIONS

*A. A. G. ALHARBI and **S.E. Khider
*Taibah University, Madina Munawara, Saudi Arabia
**Sadat Academy for Management Science, Egypt

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ABSTRACT

Comparisons between the life distribution of a new unit with that of the long term remaining life or a used unit in excess wealth order, have led us to introduce a new class of life distributions. This is the new better than used in excess wealth (NBUW). The relation of this class to other classes of life distributions, closure properties under three reliability operations are discussed. Stochastic comparisons of the excess life time at different times of a renewal process when the inter–arrival times belong to the NBUW class are established. We also provide a simple argument based on stochastic order that the family of the NBUW distributions is closed under the formation of parallel systems in case of independent but not identically distributed components. We add a new result when the stochastic comparisons are given in terms of total time on test transform ordering. Also we show that if the inter-arrival times belong to the NBUW class, then excess life time at t can be compared with the excess life time at time 0.

1- INTRODUCTION

Stochastic orderings between probability distributions play a fundamental role in probability and statistics, and have been studied by many authors (see, for example Shaked and Shanthikumar (1994) and Belzunce *et al.* (2001), among others). In reliability theory it has been found useful to define non–parametric classes of lifetime distributions by stochastic comparisons of the survival function of the residual lifetime of a used unit with the survival of the lifetime of a new one. For example, if X denotes the random (non-negative) lifetime of a unit with the life distributions F and survival function $\overline{F}$, then X is said to be NBU if

$$\overline{F}(x+t) \le \overline{F}(t)\ \overline{F}(x) \qquad \textit{for}\ \text{all}\ t \ge 0$$

For definitions of such classes of life distributions, see Barlow and Proschan (1981), for NBUE, Deshpande *et al.*, (1986) for NBU (2), Coa and Wang (1991) for NBUC. Other aging classes have been defined in reliability theory by stochastic comparisons in other criteria.

In this paper, comparisons between the life distributions of a new unit with that of the long term remaining life of a used unit in the excess wealth (see, e.g. Kayed and Ahmed (2003) for a justification of the

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name) ordering have led us to introduce a new class of life distribution i.e, the class (NBUW). In Section 2, we give definitions, notation and basic facts used through the paper. Closure properties of this class under convolutions and formation of coherent structures are studied in Section 3. In Section 4, we add a new result when the stochastic comparisons are given in terms of total time on test transform ordering. Also, we show that if the inter –arrival times belong to NBUW class, then the excess lifetime at t can be compared with the excess lifetime at time 0 (that is, the time at which the first event occurs).

2- Preliminaries

In this section we present definitions, notation and basic facts used through the paper. We use "increasing" in place of "non-decreasing" and "decreasing" in place of "non-increasing". Let *X* and *Y* be two non–negative random variables with distributions function *F* and *G* and survival function $\overline{F} := 1- F$ and $\overline{G} := 1- G$ respectively. We will assume that $\overline{F}(0) = \overline{G}(0) = 1$ in all cases. Consider a distributions F, of a nonnegative random variables X, which is strictly increasing on its interval support. Let $p \in (0, 1)$ and $t \geq 0$ be two values related by $p = F(t)$ or, equivalently, by $t = F^{-1}(p)$, where F^{-1} is the right continuous inverse of *F*. Let $X_t \equiv [X-t \mid X > t]$ denote to the residual lifetime of X at time t.

Definition 2.1

For any life variable $X \geq 0$, a residual life variable X_t is a non - negative random variables representing the remaining life of X at age t. Hence if *F* is the distributions function of *X* and $\overline{F}$ is its survival function, then the survival function of X_t is given by

$$\overline{F}_t(x) = \frac{\overline{F}(x+t)}{\overline{F}(t)} \quad ; x \geq 0, t \geq 0. \tag{2.1}$$

For the definitions and the properties below see *Shaked* and *Shanthikumar* (1994) and Kochar *et al. (2002)*.

Definition 2.2

Let X and Y be two non - negative random variables, then X is said to be less than Y in the :

(a) Stochastic order (denoted by $X \leq_{st} Y$) if $E[\phi(X)] \leq E[\phi(Y)]$ for all increasing functions ϕ for which previous expectations exist

(b) Increasing convex order (denote by $X \leq_{icx} Y$) if $E[\phi(X)] \leq E[\phi(Y)]$ for all increasing convex functions ϕ for which previous expectations exist .

(c) Excess wealth ordering (denote by $X \leq_{ew} Y$) if $E[\phi(X)] \leq E[\phi(Y)]$ for all convex function for which previous expectations exist.

Theorem 2.1

Let X and Y be two non-negative random variables, with distribution functions F and G, survival functions $\overline{F}$ and $\overline{G}$, respectively , then :

(a) $X \leq_{icx} Y$ if , and only if ,

$$\int_x^\infty \overline{F}(u)\,du \leq \int_x^\infty \overline{G}(u)\,du, \quad for\ all \quad x \geq 0,$$

(b) $X \leq_{ew} Y$ if, and only if,

$$\int_{F^{-1}(p)}^\infty \overline{F}(x+t)dx \leq \int_{F^{-1}(p)}^\infty \overline{G}(x)dx, \quad forall \quad p \in (0,1)$$

For the previous orders we have the following relationships

$$X \leq_{st} Y \Rightarrow X \leq_{ew} Y \Rightarrow X \leq_{icx} Y$$
$$\Downarrow$$
$$E(X) \leq E(Y).$$

For the sake of completeness we give the definitions of the non-parametric aging classes that we have pointed out in this paper (see, Barlow and Proschan (1981) for NBUE, Deshpande *et al.* (1986) for NBU(2), Cao and Wang (1991) for NBUC, Belezunce *et al.* (2001) for NBU_{Lt}) .

Definition 2.3

Let X be a nonnegative random variable with distribution function F, we say that:

(a) X (or F) is new better than used (denoted by $X \in NBU$) if

$$X_t \leq_{st} X \quad \text{for all } t \geq 0;$$

(b) X (or F) is new better than used in the concave order (denoted by X ∈ NBU(2)) if

$$X_t \leq_{icv} X \quad \text{for all } t \geq 0;$$

(c) X (or F) is new better than used in the Laplace order (denoted by X∈ NBU(2) $_{Lt}$) if

$$X_t \leq_{Lt} X \quad \text{for all } t \geq 0.$$

Following these ideas, we have defined of the new better than used in excess wealth ordering (*NBUW*) class of life distributions.

Theorem 2.2

F∈ ***NBUW* if**

$$\int_{F^{-1}(p)}^{\infty} \overline{F}(x+t)dx \leq \overline{F}(t) \int_{F^{-1}(p)}^{\infty} \overline{F}(x)\,dx, \qquad \text{forall } x,\ t\geq 0 \text{ and } p\in(0,1) \qquad (2.2)$$

Proof

The proof is obvious from (2.1) and (2.2) and hence is omitted . The following theorem gives the relationship between the *NBUW* class and the two well known classes *NBU* and *NBUC* .

Theorem 2.3

(i) If X *NBU* then X is *NBUW* .

(ii) If X is *NBUW* then X is *NBUC*.

Proof

(i) From (2.1) we have X is NBU $\Rightarrow X_t \leq_{st} X$, for all $t \geq 0$

$\Rightarrow X_t \leq_{ew} X$, for all $t \geq 0$

$\Rightarrow X_t$ is *NBUW* .

The Proof of (ii) is similar.

3. Preservation properties

The three operations discussed in the theory of reliability are convolution , mixture of distributions and coherent systems .

3.1 Preservation under convolution

Convolution of certain life distributions have been given great attention in the literatures. It has been shown that the classes *NBUE*, *NBUC* and *NBU* (2) are closed under this operation (Barlow and Proshan (1981), Cao and Wang

(1991) Li and Kochar (2001). In the next theorem we establish the closure property of NBUW class under the convolution operation .

Theorem 3.1

Suppose that F_1 and F_2 are two independent *NBUW* life distributions, then their convolution is also NBUW .

Proof

The survival function of the convolution of two life distributions F_1 and F_2 is

$$F(y)=\int_0^\infty \overline{F}_1\ (y-z)\ dF_2\ (z).$$

By Fubini theorem and NBUW property, we have, for all $t \geq 0$, $x \geq 0$ and $p\in(0,1)$,

$$\int_{F^{-1}(p)}^{\infty}\overline{F}(y+t)\,dx=\int_{F^{-1}(p)}^{\infty}\int_0^\infty \overline{F}_1(y+t-z)\,dF_2(z)\ dy$$

$$=\int_0^\infty\int_{F^{-1}(p)}^{\infty}\overline{F}_1(y+t-z)\,dx\,dF_2\ (z)$$

$$\leq\int_0^\infty \overline{F}_1(t)\quad\int_{F^{-1}(p)}^{\infty}\overline{F}_1(y-z)\ dx\ dF_2(z)$$

$$=\int_0^\infty \overline{F}_1(t)\quad\int_{F^{-1}(p)}^{\infty}\ \int_0^\infty \overline{F}_1(y-z)\ dF_2\ (z)dy$$

$$\leq\overline{F}(t)\quad\int_{F^{-1}(p)}^{\infty}\overline{F}(y)dy,$$

where the last inequality follows from the fact that the convolution of two independent nonnegative random variables is stochastically larger than each of them. This proves that F is also *NBUW*.

3.2 Preservation of parallel systems

Consider an n-component system, and let the vector $X = (x_1, \ldots, x_n)$ be such that $X_i = 1$ if component i functions and $X_i = 0$ if it fails, for $i = 1,\ldots,n$. Let also $\psi(X) = 1$ if the system functions and $\psi\ (X) = 0$ otherwise. The structure functions ψ of the system is said to be coherent if it is non-decreasing and not constant in any X_i (see Barlow and Proschan (1981) for details).

Assume the component lifetimes of a coherent system to be non-negative random values X_i. In this case the random lifetime T_n of the system can be expressed as a function t_n $(x_i,..., x_n)$ of the component lifetimes. Such functions are called coherent life functions. For example, if the system consists of two active components acting in parallel, with random lifetimes X_1 and X_2, then the lifetime of the system can be expressed as $T_2 = \max(X_1, X_2)$.

We say that the class *NBUW* is closed with respect to formation of coherent systems (or a subclass of coherent system., i.e., parallel or series) if the distribution of the system lifetime is a member of *NBUW* whenever the distribution of the components lifetimes are members of NBUW. Actually. It is a well known fact that some aging notations are preserved under formation of general coherent systems of mutually independent components (see Barlow and Proschan (1981) for NBU and IFRA, Loh (1984) for NBAFR and Pellerey (1993) for NBUFR), while others are preserved under formation of simple parallel or series system (see Barlow and Proschan (1981) for IFR, Abouammoh and EL-Neweihi (1986) for DMRL and NBUE and Hendi and Mashhour., (1993) and Pellerey and Petekos (2002) for NBUC).

In the following, we prove that the NBUW class is closed under formation of parallel systems. with independent but not necessarily identically distributed components. We need the following lemma.

Lemma 3.1.

Let $X = (X_1, X_2,..., X_n)$ and $Y = (Y_1, Y_2, ..., Y_n)$ be two n-dimensional random vectors. If $X \leq_{ew} Y$ and $g : R^n \to R$ is any increasing and convex function, then

$$g(X_1, X_2, ..., X_n) \leq_{ew} \quad g(Y_1, Y_2, ..., Y_n).$$

Observing that vector dominance in the excess wealth ordering is componentwise dominance, the proof of this lemma is similar to the univariate counterpart of Theorem 4.2 of Kochar *et al.* (2002) and is omitted.

Theorem 3.2.

Let T be component wise concave coherent life function of a system with n components. If the components lifetime distribution X_i, i=1, ... , n are independent and *NBUW*, then the system life itself $T_n = t_n(X_1, ..., X_n)$ has a *NBUW* distribution.

Proof.

According to the definition of NBUW, we need to prove that

$$[\, T_n - t \mid T_n > t \,] \leq_{ew} T_n \quad \text{for every } t \geq 0.$$

Observe that (see Pellerey and Petakos (1999))

$$[\, T_n - t \mid T_n > t \,] \leq_{st} t_n (\{X_1 - t \mid X_1 > t\}, \dots, \{X_n - t \mid X_n > t\}). \qquad (3.1)$$

Moreover. the NBUW property of the X_i and Lemma 3.1, imply that

$$t_n ([\, X_1 - t \mid X_1 > t \,], \dots, (X_n - t \mid X_n > t \,]) \leq_{ew} t_n (X_1, \dots, X_n) = T_n. \qquad (3.2)$$

Since usual stochastic ordering implies the excess ordering (see Kochar et al. (2002), relations (3.1) and (3.2) in conjunction with the transitivity property lead to

$$[\, T_n - t \mid T_n > t \,] \leq_{ew} T_n \qquad \text{for every fixed } t \in R^+ ,$$

and the proof is completed.

Corollary 3.1.

Suppose that $\overline{F}_i$, (i=1, ... , n) are survival functions of n mutually independent NBUW elements $X_1, \dots, X_n$, respectively. If all of them have finite means, then the parallel system of these units is NBUW as well.

Proof.

Since the $\max_{i \in I} X_i$, have a component wise convex function, using Theorem 3.2, it is easy to verify that $\max_{i \in I} X_i$ belongs to NBUW.

4 -Stochastic comparisons of excess lifetime renewal processes

Let us consider a renewal process with independent and identically distributed non-negative inter-arrival times X_i, with common distribution *F* and *F* (0) =0. Let $S_0 = 0$ and $S_k = \sum_{i=1}^{k} X_i$, and consider the renewal counting process N(t) = Sup {n : $S_n \leq t$}. Several papers have investigated some characteristics of the renewal process related to aging properties of *F*.

For example, Barlow and Proschan (1981), Shaked and Zhang (1992) Brown (1980, 1981), and Chen (1994) have investigated the relationship between the behavior of the renewal function M(t) = E(N(t)) and aging property of *F*. Some other results are given for the excess lifetime at time t > 0, that is,

$\gamma(t)$ $_{=S_{n(t)+1}}$ -1, which is the time of the next event a time t. (see Barlow and Proschan, (1981) p.147 for definition of γ (t)).

Some examples of such results are the following:

(i) Chen (1994) showed that (a) if $\gamma(t)$ is stochastically decreasing in $t \ge 0$. then $F \in$ NBU and (b) if $E[\gamma(t)]$ is stochastically decreasing in $t \ge 0$, then $F \in$ *NBUE*.

(ii) Li and Yam (2003) showed that, if γ (t) is decreasing in $t \in\ge 0$, in the increasing convex order, then $F \in$ NBUC.

(iii) Li and Kochar (2001) showed that, if γ (t) is decreasing in $t \ge 0$ in the increasing concave order, then $F \in$ *NBU(2)* .

(iv) Belzunce et al. (2001) showed that, if γ (t) is decreasing in $t \ge 0$ in the Laplace order, then $F \in NBU_{Lt}$.

Next we show a similar result for excess wealth order and NBUW aging class.

Theorom 4.1.

If γ (t) is decreasing in t for all $t \ge 0$ in the excess wealth order, then $F \in$ NBUN.

Proof

Let $u(t, x) = P(\gamma(t) \ge x)$, for all x, $t \ge 0$

It follows that (see Karlin and Taylor (1975, P. 1993)

$$u(t,x) = \overline{F}(x+t) + \int_0^t u(t-s,x)\,dF(s) \qquad (4.1)$$

From (2.2) and (4.1), we have for all $p \in (0, 1)$

$$\int_{F^{-1}(P)}^{\infty} u(t,x)dx = \int_{F^{-1}(P)}^{\infty} \overline{F}(x+t)dx + \int_{F^{-1}(P)}^{\infty}\int_0^t u(t-s,x)dF(s)dx$$

$$= I_1 + I_2$$

Since γ (t) is decreasing in t in excess wealth ordering for all $t \ge 0$ and $p \in (0,1)$, we have from (2.2)

$$I_2 = \int_{F^{-1}(P)}^{\infty}\int_0^t u(t-s,x)dF(s)dx$$

$$= \int_{0}^{t} \int_{F^{-1}(P)}^{\infty} u(t-s,x)dF(s)dx$$

$$\geq \int_{0}^{t} \int_{F^{-1}(P)}^{\infty} u(t-s,x)dF(s)dx$$

$$= \int_{F^{-1}(P)}^{\infty} \int_{0}^{t} u(t-s,x)dF(s)dx$$

$$= F(t) \int_{F^{-1}(P)}^{\infty} u(t-x)dx\,.$$

Thus, we have, for all p∈ (0,1),

$$\int_{F^{-1}(P)}^{\infty} u(t-s,x)dF(s)dx \geq \int_{F^{-1}(P)}^{\infty} \overline{F}(x+t)F(t)dx \int_{F^{-1}(P)}^{\infty} u(t,x))dx.$$

That, is for all p∈ (0,1).

$$\int_{F^{-1}(P)}^{\infty} \overline{F}(x+t)dx \leq \overline{F}(t) \int_{F^{-1}(P)}^{\infty} u(t,x)dx \qquad (4.2)$$

In addition, for all t ≥ 0, X = γ (0) the underlying distribution of the interarrivals of the renewal process is stochastically larger than the random variable γ (t), thus, for all p ∈ (0, 1) and t ≥ 0,

$$\int_{F^{-1}(P)}^{\infty} U(x+t)dx = \int_{F^{-1}(P)}^{\infty} P(\gamma(t) \geq x)dx \geq \int_{F^{-1}(P)}^{\infty} \overline{F}(x)dx. \qquad (4.3)$$

By (4.2) and (4.3), we have for all t > 0 and p∈ (0, 1),

$$\int_{F^{-1}(P)}^{\infty} \overline{F}(x+t)dx \leq \overline{F}(t) \int_{F^{-1}(P)}^{\infty} \overline{F}(x))dx.$$

and therefore F ∈ *NBUW*.

Whereas these results give sufficient conditions for the aging property of *F*, in practical situations it would be more interesting to derive some properties for γ (t) from the aging property of *F*. In fact, given a renewal process it is more feasible to study it if *F* has some aging property than if γ (t) has some of the previous properties.

A result in such a direction is the following one:

(a) Barlow and Proschan (1981. p. 169) showed that, if F ∈ NBU, then γ (t) $\leq_{st}$ γ (0), for all t ≥ 0.

(b) Belzunce et al. (2001) showed that:

(i) if F ∈ *NBUE*, then E[γ (t)] ≤ E[γ (0)], for all t ≥ 0 ;

(ii) if F ∈ NBUC, then γ (t) $\leq_{cx}$ γ (0), for all t ≥ 0 ;

(iii) if F ∈ NBU(2), then γ (t) $\leq_{icv}$ γ (0), for all t ≥ 0 ;

(iv) if $F \in NBL_{Lt}$, then $\gamma(t) \le \gamma(0)$. for all $t \ge 0$.

Next we give a similar result for the NBUW class.

Theorem 4.2.

Given a renewal process as above, if $F \in$ NBUW, then

$$\gamma(t) \le_{ew} \gamma(0), \text{ for all } t \ge 0.$$

Proof

For the equality,

$$p(\gamma(t) \ge u) = \overline{F}(t+u)\int_0^t \overline{F}(t-s+u)dM(s),$$

see Barlow and Proschan (1981, p. 168)), given $t \ge 0$ and $p \in (0\ ,1)$, we have

$$\int_{F^{-1}(P)}^{\infty} P(\gamma(t) \ge u)du = \int_{F^{-1}(P)}^{\infty} \overline{F}(t+u)du + \int_{F^{-1}(P)}^{\infty}\int_0^t \overline{F}(t-s+u)dM(s)du$$

and given that $F \in$NBUW, we have the inequality

$$\int_{F^{-1}(P)}^{\infty} P(\gamma(t) \ge u)du \le \overline{F}(t)\int_{F^{-1}(P)}^{\infty} \overline{F}(u)du + \int_{F^{-1}(P)}^{\infty} \overline{F}(u)\overline{F}(t-s)dM(s)du$$

$$= \int_{F^{-1}(P)}^{\infty} \overline{F}(u)du\left(\overline{F}(t) + \int_0^t \overline{F}(t-s)dM(s)\right)$$

$$= \int_{F^{-1}(P)}^{\infty} \overline{F}(u)du.$$

and therefore $\gamma(t) \le_{ew} \gamma(0)$

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عائلةالحياة الجديدة أفضل من المستخدم في ترتيب الثروة

عبد الغني أبو الحسن الحربي - سليم الزناتي خضر

يعنى هذا البحث بتعريف ودراسة خصائص عائلة حياة جديدة مبنية على مقارنة توزيع حياة وحدة جديدة بأخرى عاشت لفترة زمنية محددة ودراسة ما تبقي من عمرها.
هذه المقارنات عادة ما تظهر عندما تقارن الثروات الموزعة في مجتمع ما . أطلقنا على العائلة المستخدمة اسم الجديد أفضل من المستخدم في ترتيب توزيع الثروة (NBUW) . تمت دراسة علاقة هذه العائلة بالعائلات الأخرى المعرفة فيما سبق هذا البحث من أبحاث وكذلك تمت دراسة خصائص العائلة فيما يتعلق بعمليات الموثوقية الثلاث الرئيسية (اللف والخلط والنظم).
كما تم إجراء مقارنات إحتمالية بين عمليات تجديد مختلفة تخضع أزمنتها لتوزيع العائلة وكذلك أجريت عمليات مقارنة عشوائية لزمن الاختبار الكلي (Total Time on Test) مع مقارنة زمن التجاوز(Excess Time) لوحدات عمرهاt بوحدات أخرى جديدة (عمرها صفر).