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BOUNDARY VALUE PROBLEM FOR ODD-ORDER DIFFERENTIAL-OPERATIONAL EQUATIONS

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ABSTRACT

In this article we prove the existence and uniqueness of strong solution of boundary value problem for odd-order differential-operational equations with variable operational-coefficient domains. The proof is based on an energy inequality and the density of the range of the operator generated by the problem.

1. INTRODUCTION

Let $\Omega=(0,T_1)\cup(T_1,T)$ where $(0,T_1)$ and (T_1,T) are bounded intervals of values of real variable t and H be a Hilbert space with the norm $|\bullet|_0$ and the scalar product $(\bullet,\bullet)_0$. On $\Omega=(0,T_1)\cup(T_1,T)$, we consider the following boundary- value problem

$$\Im v = c_{2n+1}\frac{d^{2n+1}v}{dt^{2n+1}}+c_{2n}\frac{d^{2n}v}{dt^{2n}}+\sum_{k=0}^{n}A_k(t)\frac{d^k v}{dt^k}+A(t)v=f(t) \qquad (1.1)$$

Here v and f are functions of t with values in the Hilbert space H; $A_k\ (k=0,1,\cdots,n)$ are bounded linear operators on H. A is a closed linear operator on H with an everywhere-dense domain $D(A)$ depending on t, and $c_{2n+1}(t)$ and $c_{2n}(t)$ are scalar functions such that

$$c_{2n+1}(t)=\begin{cases}1, & 0<t<T_1,\\ 0, & T_1<t<T,\end{cases} \qquad c_{2n}(t)=\begin{cases}0, & 0<t<T_1,\\ 1, & T_1<t<T,\end{cases} \qquad (1.2)$$

The conditions imposed on $A(t)$, is given by the inequality

$$(-1)^n\,\mathrm{Re}(A(t)w,w)_0\geq 0, \qquad \forall w\in D(A(t)),\quad t\in(0,T) \qquad (1.3)$$

and it is satisfied for each $t\in\Omega$, and the conjugate operator $A^*(t)$ satisfies the analogous inequality.

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Together with the boundary value problem (1.1), the following boundary conditions must be satisfied:

$$\frac{d^k v}{dt^k}\Big|_{t=0}=\frac{d^l v}{dt^l}\Big|_{t=T}=0, \qquad 0\le k\le n, \qquad 0\le l\le n-1. \qquad (1.4)$$

Extra conditions are imposed on $v(t)$ at the points of discontinuity of $c_{2n+1}(t)$ and $c_{2n}(t)$; the conditions are

$$\frac{d^i v}{d\,t^i}\Big|_{t=T_1-0}=\alpha_i\frac{d^i v}{d\,t^i}\Big|_{t=T_1+0} \qquad 0\le i\le 2n-1 \qquad (1.5)$$

Where $t=T_1-0$ and $t=T_1+0$ indicated limits at $t=T_1$ from the left and right respectively, and $(i+1)\,\alpha_{2n-(i+1)}\ \alpha_i=1$.

V. I. Korzyuk and N. I Yurchuk [16] studied problem (1.1), with the boundary conditions (1.4) in the special case when $n=1$. Two–point boundary value problem have been investigated for differential - operational equations in [15-24]. Cauchy problem for three-order hyperbolic differential-operational equation studied in [17].The abstract generalization of boundary value problem in the case of cylindrical region, investigated in [16], for correspond differential hyperbolic equation with initial– boundary value problems studied in [15], which is described the distribution of the linear acoustic waves in dispersion media [11]. For the case of many-point boundary value problem for some differential-operational equations, a priori energy inequality are established in [2-5] and a solvability and properties of the solution of many – point boundary value problem are given in [3]. Our problem is the study of the case of many – point boundary value problem for some differentia-operational equations of odd order. We shall prove that problem (1.1), (1.4-5) has a unique solution depending continuously on the operation-valued coefficients in the equation, the proof is based on an energy inequality bounds and the fact that the range $R(\Im)$ of $\Im$ is dense, the energy inequality is proved by the standard method. To prove that $R(\Im)$ is dense we use the abstract smoothing operators $A_\varepsilon^{-1} = (I + \varepsilon A)^{-1}$, $\varepsilon \geq 0$. The continuous dependence of the solution on the coefficients is proved by using consequences of the Banach-Steinhaus theorem.

2. A Priori Energy Inequality and its Consequences

We next describe the notation we use and some basic results needed from [2].

Let $D(\Im)$ be the set of all integrable functions $v \in L_2(\Omega, H)$, such that

$$v(t) \in D(A),\ Av \in L_2(\Omega, H),\ \frac{d^{2n}v}{dt^{2n}} \in L_2(\Omega, H),\ \frac{d^{2n-1}v}{dt^{2n-1}} \in L_2((0,T_1), H),$$

and v satisfies conditions (1.4) and conditions (1.5).

From (1.1), if $v \in D(\Im)$, and $A_k(t) \in L_\infty(\Omega, B(H,H))$, then $\Im v \in L_2(\Omega, H)$, here by $B(H,H)$ we mean Banach space of bounded linear operators from H to H. We write $\|\bullet\|$ and $(\bullet,\bullet)$ the norm and the scalar production in $L_2(\Omega, H)$, and $\|\bullet\|_\infty$ the norm in $L_\infty(\Omega, B(H,H))$.

If v satisfies the conditions (1.4), then

$$\int_0^{T_1} \left|\frac{d^k v}{dt^k}\right|_0^2 dt \leq \frac{T_1^{2(n-k)}}{\alpha^2(n,k)} \int_0^{T_1} \left|\frac{d^n v}{dt^n}\right|_0^2 dt$$

$$\int_{T_1}^{T} \left|\frac{d^k v}{dt^k}\right|_0^2 dt \leq \frac{(T-T_1)^{2(n-k)}}{\alpha^2(n,k)} \int_{T_1}^{T} \left|\frac{d^n v}{dt^n}\right|_0^2 dt \tag{2.1}$$

Where $0 \leq k \leq n$, $\quad \alpha^2(n,k) = 2(n-k)!\,(n-k-1)!\,(2n-2k-1)$.

Theorem 2.1. In equation (1.1), let $A_k(t) \in L_\infty(\Omega, B(H,H))$, and suppose that

$$\sum_{k=0}^{n} \|A_k\|_\infty \frac{T^{2n-k+1}}{\alpha(n,0)\,\alpha(n,k)} < 2n+1, \qquad 0 < t < T_1$$

$$\sum_{k=0}^{n} \|A_k\|_\infty \frac{(T-T_1)^{2n-k}}{\alpha(n,0)\,\alpha(n,k)} < 1, \qquad T_1 < t < T \tag{2.2}$$

Then for each $v \in D(\Im)$ the following inequality holds:

$$\|v\|_{n,\psi_A} \le M \sup_{w} \frac{\left| \int_0^T \psi(t)\,(\Im v, w)_0\, dt \right|}{\|w\|_{n,\psi_A}} \tag{2.3}$$

Where

$$\|v\|^2_{n,\psi_A} = \int_0^T \left(\left| \frac{d^n v}{dt^n} \right|_0^2 + (-1)^n \psi(t) \operatorname{Re}(\Im v, v)_0 \right) dt \tag{2.4}$$

and $\psi(t) = T_1 - t$ for $0 < t < T_1$ and $\psi(t) = 1$ for $T_1 < t < T$.

Proof. Integrating by parts and using the boundary conditions (1.4) and (1.5) give

$$(-1)^n 2\operatorname{Re}\left[\int_0^{T_1} \left(\frac{d^{2n+1} v}{dt^{2n+1}}, (T_1 - t)\, v\right)_0 dt + \int_{T_1}^{T} \left(\frac{d^{2n} v}{dt^{2n}}, v\right)_0 dt \right]$$
$$= (2n+1)\int_0^{T_1} \left| \frac{d^n v}{dt^n} \right|_0^2 dt + 2\int_{T_1}^{T} \left| \frac{d^n v}{dt^n} \right|_0^2 dt. \tag{2.5}$$

Now integrating $(-1)^n 2\operatorname{Re}\psi(t)\,(\Im v, v)_0$ over Ω, applying (1.1), (2.1), (2.2) and (2.5) and making some elementary transformations, to get

$$\|v\|^2_{n,\psi_A} \le (-1)^n M_T \operatorname{Re} \int_{\Omega} \psi(t)\,(\Im v, v)_0\, dt\,. \tag{2.6}$$

This inequality shows that the operator $\Im$ on an appropriate space satisfies an inequality of the form (1.2). For the right side in (2.6) we use the upper bound

$$\sup_{w} \frac{\left|\int_{\Omega} \psi(t)\,(\Im v, w)_0\, dt\right|}{\|w\|_{\pi;\psi_A}} \|v\|_{\pi;\psi_A} \tag{2.7}$$

and get (2.3), this proves theorem 2.1.

Remark1.1. Inequality (2.3) implies the simpler but weaker inequality

$$\|v\|_{\pi;\psi_A} \le M \|\Im v\| \tag{2.8}$$

We begin by introducing the required spaces, let E be the solution space, which is the completion of $D(\Im)$ in the norm $\|v\|_{\mathrm{E}}$ equal to $\|v\|_{\pi;\psi_A}$. Let F denotes the space of right side of equation (1.1), which is the completion of $L_2(\Omega, H)$, in the norm $\|f\|_F$ equal to the norm

$$\|v\|_{\pi;\psi_A} \le M \sup_{w} \frac{\left|\int_0^T \psi(t)\,(\Im v, w)_0\, dt\right|}{\|w\|_{\pi;\psi_A}}$$

This can be written in the form

$$\|v\|_{\mathrm{E}} \le M \|\Im v\|_F, \quad v \in D(\Im) \tag{2.9}$$

Let $\Im$ be the operator from E to F with domain $D(\Im)$ defined by equation (1.1).

Corollary 2.1 The operator $\Im : \mathrm{E} \to F$ has a closure.

Proof. By virtue of a known criterion for operators on Banach spaces to have closure, [14], it is sufficient to prove that, if $D(\Im) \ni v_k \to 0$ in E and $\Im v_k \to f$ in F for $k \to \infty$, then $f = 0$.

For $w \in \overset{*}{D}(\Im) \subset \mathrm{E}$, satisfying the boundary conditions (1.4) and (1.5) then after integrating by parts and taking the limit the following relation holds

$$f(w) = \lim_{k\to\infty} \int_0^T (\Im v_k, \psi w)_0\, dt = \lim_{k\to\infty} \int_0^T (A w_k, \psi w)_0\, dt$$

The inequality

$$\mathrm{Re}[(Aw_k,\psi w)_0+(w_k,A\psi w)_0]\le 2\sqrt{\mathrm{Re}(Aw_k,w_k)_0}\ \sqrt{\mathrm{Re}(A\psi w,\psi)_0}\ .$$

Implies that $\lim_{k\to\infty}\int_0^T (Aw_k,\psi w)_0\,dt=\lim_{k\to\infty}\int_0^T (w_k,A\psi w)_0\,dt=0;$

hence $f(w)=0$, the set of functions w is complete in E; hence $f=0$. Let $\overline{\Im}$ be the closure of $\Im$. A function $v\in\mathrm{E}$ is in $D(\overline{\Im})$, if there is a sequence $v_k\in D(\Im)$ and an element $f\in F$ such that $v_k\to v$ in E and $\Im v_k\to f$ in F. Then

$$\overline{\Im}v=f=\lim_{k\to\infty}\Im v_k.$$

Definition. A solution of the equation

$$\overline{\Im}v=f,\quad f\in F \tag{2.10}$$

is said to be a strong solution of the corresponding boundary-value problem. By taking the limit we can extend the inequality (2.9) to apply to $v\in D(\overline{\Im})$ that is,

$$\|v\|_{\mathrm{E}}\le M\|\overline{\Im}v\|_F,\quad v\in D(\overline{\Im}). \tag{2.11}$$

Corollary 2.2 The range $R(\overline{\Im})$ of $\overline{\Im}$ is closed in F, $R(\overline{\Im})=\overline{R(\Re)}$ and there is a bounded inverse operator $(\overline{\Im})^{-1}$ on the range of the closures, $R(\overline{\Im})$. This establishes that, to prove that (1.1) with the boundary conditions (1.4-5) has a strong solution for $f\in F$, it is sufficient to prove that the/ range $\overline{R(\Im)}=F$, i.e. to prove that the range $R(\Im)$ of $\Im$ is everywhere dense in F.

3. Density of the range $R(\Im)$ of $\Im$

Let $\tilde{A}=(-1)^n A+\lambda I$ where $\lambda>0$ is an arbitrary fixed number. We use the Banach space of continuous linear mappings of the Banach space B_1 into the Banach space B_2, which we denote by $B(\mathrm{B}_1,\mathrm{B}_2)$.

Theorem 3.1

If theorem 2.1 is satisfied, $\dfrac{d^sA}{dt^s}\tilde{A}^{-1}\in L_\infty(\Omega,B(H,H)),\quad 0<s\le n+1,$

exist, and $w\in D(A)$, $\dfrac{d^sA(t)}{dt^s}w\Big|_{t=T_1\pm 0}=0$, then the range $R(\Im)$ of $\Im$ is

dense in F.

Proof. The part $\sum_{k=0}^{n} A_k(t)\frac{d^k}{d\,t^k}$ of the operator $\Im$ in equation (1.1) is bounded on E into F, hence by standard method of construction by means of a parameter we can prove that, if the theorem holds for $A_k = 0$, it holds in the general case. Thus we precede under assumption that $A_k = 0$ in (1.1).

The space F is reflexive; hence, by virtue of a corollary of the Hahn-Banach theorem, we can prove that $R(\Im)$ is dense in F by stablishing that, if

$$\int_{\Omega}(\Im v,\psi(t)w)_0\,dt = 0 \quad \text{for all } v \text{ in } D(\Im) \text{ and all } w \text{ in } F. \tag{3.1}$$

Then $w = 0$. From equation (1.1); we get

$$\int_0^{T_1}(T_1-t)\left(\frac{d^{2n+1}v}{dt^{2n+1}},w\right)_0 dt + \int_{T_1}^{T}\left(\frac{d^{2n}v}{dt^{2n}},w\right)_0 dt = -\int_0^{T}(\psi(t)(Av,w)_0\,dt. \tag{3.2}$$

Integrating by parts and using the boundary conditions (1.4) and (1.5) for v and conditions (1.4) and (1.5) with $0 \le i \le n$ for w give

$$\int_0^{T_1}\left(\frac{d^{n+1}v}{dt^{n+1}},\frac{d^n(T_1-t)w}{dt^n}\right)_0 dt - \int_{T_1}^{T}\left(\frac{d^{n+1}v}{dt^{n+1}},\frac{d^{n-1}w}{dt^{n-1}}\right)_0 dt \tag{3.3}$$

$$= (-)^{n+1}\int_0^{T}(\psi(t)(Av,w)_0\,dt$$

By applying a limit process we can prove that this relation holds for $v \in L_2(\Omega, H)$ such that

$\frac{d^{n+1}v}{dt^{n+1}} \in L_2(\Omega, H)$, $v(t) \in D(A(t))$, $Av \in L_2(\Omega, H)$ and v satisfies conditions (1.4) and (1.5) for $0 \le i \le n$. We also need a family of restrictions, uniform with respect to ε on the family of operators

$A_\varepsilon^{-1}=(I+\varepsilon\widetilde{A})^{-1}$, $\varepsilon \geq 0$ is a parameter and the operator $\widetilde{A}$ is defined in the beginning of this section, and it is an analog of a smoothing operator (or an averaging operator in another terminology). The operators A_ε^{-1} were first used for smoothing in [9]; they have the following properties. If $\varepsilon \neq 0$, then $A_\varepsilon^{-1}g$ takes values in $D(A)$ for each $g \in H$. If $w \in D(A)$, then $A A_\varepsilon^{-1} w = A_\varepsilon^{-1} A w$. The norms of $\varepsilon\widetilde{A}A_\varepsilon^{-1}=A_\varepsilon^{-1}-I$ are bounded by unity uniformly with respect to ε and t, and for each $v \in L_2(\Omega, H)$ with range in $D(A)$, we have $\|\varepsilon\widetilde{A}A_\varepsilon^{-1}w\| = \| A_\varepsilon^{-1}w - w \| \to 0$ for $\varepsilon \to 0$.

By virtue of a corollary to the Banach-Steinhaus theorem, this limiting property also holds for $w \in L_2(\Omega, H)$.The differentiability properties of A imply that the operator A_ε^{-1} also have derivatives up to order $n+1$, whose values tend to zero when $\varepsilon \to 0$ for each $w \in L_2(\Omega, H)$.The operators $(A_\varepsilon^{-1})^* = (I-\varepsilon\widetilde{A}^*)^{-1}$ have the same properties.

Putting $v = A_\varepsilon^{-1} g$ in (3.3), where $g \in L_2(\Omega, H)$, $\frac{d^{n+1}g}{dt^{n+1}} \in L_2(\Omega, H)$, and g satisfies conditions (1.4) and conditions (1.5) for $0 \leq i \leq n$ gives

$$\int_0^T \left(\frac{d^{n+1}g}{dt^{n+1}}, W_\varepsilon\right)_0 dt = (-)^{n+1}\int_0^T (\psi(t)(g, A^*(A_\varepsilon^{-1})^* w)_0\, dt + \Phi_\varepsilon(g,w) \quad (3.4)$$

Where

$$W_\varepsilon(t) = (A_\varepsilon^{-1})^* \frac{d^n((T_1-t)w)}{dt^n} \qquad \text{for} \quad 0 < t < T_1 .$$

$$W_\varepsilon(t) = -(A_\varepsilon^{-1})^* \frac{d^{n-1}w}{dt^{n-1}} \qquad \text{for } T_1 < t < T \text{, and}$$

$$\Phi_\varepsilon(g,w) = -\sum_{r=0}^{n+1} C_{n+1}^r \Big[\int_0^{T_1} \Big(\frac{d^s A_\varepsilon^{-1}}{dt^s} \frac{d^{n+1-s}g}{dt^{n+1-s}}, \frac{d^n((T_1-t)w)}{dt^n}\Big)_0\, dt - \int_{T_1}^{T} \Big(\frac{d^s A_\varepsilon^{-1}}{dt^s} \frac{d^{n+1-s}g}{dt^{n+1-s}}, \frac{d^{n-1}w)}{dt^{n-1}}\Big)_0\, dt \Big] \tag{3.5}$$

It follows that $\left|\Phi_\varepsilon(g,w)\right| \le C(\varepsilon,w)\left\|\frac{d^n g}{dt^n}\right\|$; hence (3.4) implies that the mapping

$$\frac{d^n g}{dt^n} \quad \to \quad \int_0^T \Big(\frac{d^{n+1}g}{dt^{n+1}}, W_\varepsilon\Big)_0\, dt = \int_0^T \Big(\frac{d}{dt}\Big(\frac{d^n g}{dt^n}\Big), W_\varepsilon\Big)_0\, dt$$

is a continuous linear functional, where $\frac{d^n g}{dt^n}$ satisfies the following conditions

$$\frac{d^n g}{dt^n}\Big|_{t=0} = 0, \qquad \frac{d^n g}{dt^n}\Big|_{t=T_1-0} = \alpha_n \frac{d^n g}{dt^n}\Big|_{t=T_1+0},$$

and form a set whose closure in $L_2(\Omega, H)$ is orthogonal to the functions .

$$\psi(t) = \sum_{k=0}^{n-1} \psi_k(t)\, C_k^*, \text{ where } \psi_k(t) = \begin{cases} (t-T_1)^k & 0 < t < T_1 \\ (k+1)\alpha_{2n-(k+3)}(t-T_1)^k & T_1 < t < T \end{cases}$$

and C_k are arbitrary elements of H. From [6,22] we therefore conclude that W_ε is in the domain of the family of operator conjugate to the operator generated by the differential expression $\frac{dh}{dt}$ defined on Ω, with the boundary conditions

$$h\Big|_{t=0}=0, \qquad h\Big|_{t=T_1-0}=\alpha_n\, h\Big|_{t=T_1+0},$$

and orthogonal conditions $(h,\psi)=0$, hence

$$\frac{dW_\varepsilon}{dt}\in L_2(\Omega,H)\text{, and } W_\varepsilon\big|_{t=T}=0, \qquad W_\varepsilon\big|_{t=T_1-0}=\frac{1}{\alpha_n}W_\varepsilon\big|_{t=T_1+0}.$$

Integrating by parts in the left side of (3.4), therefore yields

$$\int_0^T\left(\frac{d^ng}{dt^n},\frac{dW_\varepsilon}{dt}\right)_0 dt=(-1)^{n+1}\int_0^T\psi(t)(g,A^*(A_\varepsilon^{-1})^*w)_0\,dt+\Phi_\varepsilon(g,w). \qquad (3.6)$$

By applying a limiting process we extend (3.6) to functions $g\in L_2(\Omega,H)$, such that $\frac{d^ng}{dt^n}\in L_2(\Omega,H)$ and g satisfies condition (1.4) and (1.5) containing derivatives up to the (n-1) th order, we next put $g=(A_\varepsilon^{-1})^*w$ in (3.6), after which the left side of the relation becomes

$$T_1\left|(A_\varepsilon^{-1})^*\frac{d^nw}{dt^n}\right|_0^2\Bigg|_{t=T_1-0}+\frac{2n+1}{2}\int_0^{T_1}\left|(A_\varepsilon^{-1})^*\frac{d^nw}{dt^n}\right|_0^2 dt$$
$$+\int_{T_1}^{T}\left|(A_\varepsilon^{-1})^*\frac{d^nw}{dt^n}\right|_0^2 dt-\Phi_{1_\varepsilon}(w,w) \qquad (3.7)$$

where $\Phi_{1_\varepsilon}(w,w)\to 0$ for $\varepsilon\to 0$, and the real part of the first term in the right side is nonpositive by virtue of (1.2). Now using (3.6), we obtain

$$\left\|(A_\varepsilon^{-1})^*\frac{d^nw}{dt^n}\right\|^2\le\Phi_{1_\varepsilon}(w,w)+\operatorname{Re}\Phi_\varepsilon((A_\varepsilon^{-1})^*w,w).$$

Since $\Phi_\varepsilon((A_\varepsilon^{-1})^*w,w)\to 0$ for $\varepsilon\to 0$, the limit of (3.7) for $\varepsilon\to 0$ is $\left\|\frac{d^nw}{dt^n}\right\|^2\le 0$. Hence $w=0$ and $R(\Im)$ is dense.

4. The Continuous Dependence of a Solution on Operators-Valued Coefficients

We now prove that the solutions of (1.1) and (1.4-5) depend continuously on $A_k(t)$ and $A(t)$. Consider sequences of operators $A_i(t)$, $i \geq 1$, and assume that for each i, the operator A_i satisfies conditions of theorem 3.1. We also introduce operator A_0, satisfying the conditions of theorem 3.1 and such that $D(A_0) \subset D(A_i)$. Let $A_{k,i}$ be a sequence of bounded linear operators. Consider the following sequence of differential-operational expression:

$$\Im_i\, v \equiv c_{2n+1}\frac{d^{2n+1}v}{d\,t^{2n+1}} + c_{2n}\frac{d^{2n}v}{d\,t^{2n}} + \sum_{k=0}^{n} A_{k,i}(t)\frac{d^k v}{d\,t^k} + A_i(t)\,v, \quad i \geq 1 \qquad (4.1)$$

And let $D(\Im_i)$ be the domain of $\Im_i$, defined as in the case of the domain $D(\Im)$ of $\Im$ but with A and A_k replaced by A_i and $A_{k,i}$. Let the space E_i be the completion of the set $D(\Im_i)$ in the norm $\|v\|_{\mathrm{E}_i}$, defined to be equal to $\|v\|_{\pi,\psi A_i}$, completing the set $L_2(\Omega, H)$ in the norm $\|f\|_{F_i} = \sup\limits_{w} \dfrac{\left|\int_\Omega (f, \psi w)_0\right|}{\|w\|_{\pi,\psi A}}$, we obtain the space F_i, we write $\overline{\Im_i}$ for the closure of the operator $\Im_i : \mathrm{E}_i \to F_i$ and E^0 and F^0 for spaces obtained from E_i and F_i when $A_i \equiv 0$. The imbedding $\mathrm{E}_i \subset \mathrm{E}^0$ and $F^0 \subset F_i$ hold together with the topologies and

$$\|v\|_{\mathrm{E}^0} \leq \|v\|_{\mathrm{E}_i}, \qquad \|f\|_{F_i} \leq \|f\|_{F^0} \qquad (4.2)$$

Theorem 3.1 implies that $(\overline{\Im_i})^{-1} \in B(F_i, \mathrm{E}_i)$, the space of all continues linear mapping of Banach space E_i into Banach space F_i, and the imbedding imply that $(\overline{\Im_i})^{-1} \in B(F^0, E^0)$, i.e., $v_i = (\overline{\Im_i})^{-1} f \in E^0$ for $f \in F^0$.

Theorem 4.1 If the operators $A_i(t)$ and $A_{k,i}(t), i \geq 0$ satisfies the conditions of theorem 3.1, $D(A_0) \subset D(A_i)$, operators $A_{k,i}(t)$ converge pointwise to A_k, and

$$\| A_i w - A_0 w \| \to 0 \text{ as } i \to \infty \qquad \text{for } w \in D(\Im_0) \subset D(\Im_i). \tag{4.3}$$

Then
$$\| v_i - v_0 \|_{E^0} \to 0 \tag{4.4}$$

Proof.

Relation (4.4) implies that the operators $(\Im_i)^{-1}$ converge pointwise to $(\overline{\Im_0})^{-1}$ in the space of all continues linear mapping of Banach space E^0 into Banach space F^0, $B(F^0, E^0)$. By virtue of a corollary of the Banach-Steinhaus Theorem, (4.4) will hold if the following two conditions are satisfied:

1. $\sup \| (\overline{\Im_i})^{-1} \| < \infty$

2. $(\overline{\Im_i})^{-1} g \to (\overline{\Im_0})^{-1} g$ for $g \in G$, where G is a dense set in F^0.

The first condition is satisfied by virtue of the inequalities

$$\| v \|^2_{E_i} \leq C \| \Im_i v \|^2_{F_i} \tag{4.5}$$

This can be proved in the same fashion as (3.4) and inequalities (4.2).

To verify that the second condition is satisfied, we put $G = R(\Im_0)$, and established that for $f \in R(\Im_0)$, we have

$$\| (\overline{\Im_i})^{-1} f - (\overline{\Im_0})^{-1} f \|_{E^0} \to 0 \tag{4.6}$$

If $f \in R(\Im_0)$, then $(\overline{\Im_0})^{-1} f = (\Im_0)^{-1} f = g \in D(\Im_l) \subset D(\overline{\Im_l})$; hence, in the light of (4.5) and (4.2)

$$\left\| (\overline{\Im_l})^{-1} f - (\overline{\Im_0})^{-1} f \right\|_{E^1} \leq C \left\| f - (\overline{\Im_l}) g \right\|_{F^1}^2 = C \left\| \Im_0 g - \overline{\Im_l}\, g \right\|_{F^1}^2$$

It follows from conditions Theorem 4.1 that the right side of this inequality tends to zero; hence (4.6) holds and this proves theorem 4.1.

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مسألة القيم الحدية ذاة الرتبة الفردية لمعادلات المؤثرات التفاضلية

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في هذا البحث تم برهان وجود ووحدانية الحل القوي لمسألة القيم الحدية ذات الرتبة الفردية لمعادلات المؤثرات التفاضلية والتي يتميز مداها بأن متغيراته ذات معاملات تأثير البرهان مبني على متباينة الطاقة وعلى كثافة المدى للمؤثر الناتج من المسألة.