Delta J. Sci. 2007, 31: 1 - 6 MATHEMATICS

GENERATING FUNCTIONS FOR THE GENERAL TRIPLE HYPERGEOMETRIC SERIES $F^{(3)}[x,y,z]$

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(Received: 1 March 2007)

Abstract: In this paper some generating functions for the general triple hypergeometric series $F^{(3)}[x,y,z]$ have been obtained by using the technique of integral operators. Some particular cases also have been derived.

1- Introduction

In 1991, Ragab, [5] defined Laguerre polynomials of two variables $L_n^{(\alpha,\beta)}(x,y)$ as follows:

$$L_n^{(\alpha,\beta)}(x,y)=\frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{n!}\sum_{r=0}^{n}\frac{(-y)^r L_{n-r}^{(\alpha)}(x)}{r!\Gamma(\alpha+n-r+1)\Gamma(\beta+r+1)},\qquad \text{... ... (1.1)}$$

where $L_n^{(\alpha)}(x)$ is the Laguerre polynomial of one variable defined by Rainville (see [6, p. 200]).

The Equation (1.1) is equivalent to the following explicit representation of $L_n^{(\alpha,\beta)}(x,y)$, given by Ragab [5]

$$L_n^{(\alpha,\beta)}(x,y)=\frac{(\alpha+1)_n(\beta+1)_n}{(n!)^2}\sum_{r=0}^{n}\sum_{s=0}^{n-r}\frac{(-n)_{r+s}\,y^r x^s}{(\alpha+1)_s(\beta+1)_r\,r!s!}.\qquad \text{...... (1.2)}$$

In terms of confluent hypergeometric function Ψ_2 defined by {[7]; p.59 (4.2)}.

$$\Psi_2(\alpha;\gamma,\gamma';x,y)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_{m+n}}{(\gamma)_m(\gamma')_n}\frac{x^m}{m!}\frac{y^n}{n!},\qquad \text{...... (1.3)}$$

we can write (1.2) as

$$L_n^{(\alpha,\beta)}(x,y)=\frac{(\alpha+1)_n\,(\beta+1)_n}{(n!)^2}\,\Psi_2\left[-n\,;\alpha+1,\beta+1\,;x\,,y\right] \quad \text{...... (1.4)}$$

The general triple hypergeometric series $F^{(3)}[x,y,z]$ is defined by {[7], p. 69 (3)}

$$F^{(3)}[x,\,y\,,z]=F^{(3)}\left[\begin{matrix}(a)::(b)\,;(b')\,;(b'')\,:\,(c)\,;\,(c')\,;(c'');\\ (e)::(g)\,;(g')\,;(g'')\,:(h)\,;\,(h')\,;\,(h'');\end{matrix}\,x,y,z\right]$$

$$=\sum_{m,n,p=0}^{\infty}\frac{((a))_{m+n+p}\,((b))_{m+n}\,((b'))_{n+p}\,((b''))_{p+m}\,((c))_m\,((c'))_n\,((c''))_p\,x^m y^n z^p}{((e))_{m+n+p}\,((g))_{m+n}\,((g'))_{n+p}\,((g''))_{p+m}\,((h))_m\,((h'))_n\,((h''))_p\,m!n!p!} \quad \text{...... (1.5)}$$

GENERATING FUNCTIONS of F(3) [x, y, z] .

In what follows, we prove the following result, to be used in our investigation later.

$$\int_0^\infty e^{-\left(z+\frac{s}{2}\right)t}\,t^b W_{k,\mu}(st)\,L_n^{(\alpha,\beta)}(xt,yt)\,dt$$

$$=\frac{\Gamma\left(b+\mu+\frac{3}{2}\right)\Gamma\left(b-\mu+\frac{3}{2}\right)s^{\mu+\frac{1}{2}}}{\Gamma(b-k+2)(z+s)^{b+\mu+\frac{3}{2}}}\;\frac{(\alpha+1)_n(\beta+1)_n}{(n!)^2}$$

$$\cdot F^{(3)}\left[\begin{matrix}b+\mu+\frac{3}{2}::b-\mu+\frac{3}{2},-n\,;-\,;-:\ -\ ;\ \ -\ ;\mu-k+\frac{1}{2};\\ b-k+2::\ \ -\ \ ;-\,;-:\alpha+1;\ \beta+1;\ \ -\ ;\end{matrix}\,\frac{x}{z+s},\frac{y}{z+s},\frac{z}{z+s}\right] \quad \text{......(2.1)}$$

To prove the result (2.1), we use the following result {[3], p. 216 (16)}:

$$\int_0^\infty e^{-pt}\,t^{A-1}W_{k,\mu}(st)\,dt$$

$$=\frac{\Gamma\left(A+\mu+\frac{1}{2}\right)\Gamma\left(A-\mu+\frac{1}{2}\right)s^{\mu+\frac{1}{2}}}{\Gamma(A-k+1)\left(p+\frac{s}{2}\right)^{A+\mu+\frac{1}{2}}}\;{}_2F_1\left[\begin{matrix}A+\mu+\frac{1}{2},\mu-k+\frac{1}{2};\\ A-k+1\end{matrix}\;\frac{p-\frac{s}{2}}{p+\frac{s}{2}}\right] \quad \text{... ... (2.2)}$$

And $\mathrm{Re}(A\pm\mu)>-\frac{1}{2}$ *and* $\mathrm{Re}(p+\frac{s}{2})>0$,

where $W_{k,\mu}(x)$ is Whittker function, defined by Erdelyi et al {[2], p.264 (5)}.

Proof of (2.1):

Let us write A for the terms on the left-hand side of (2.1). Expanding Ψ_2 into power series and changing the order of summation and integration, which is permissible due to the absolute convergence of integrals, we have

$$A=\frac{(\alpha+1)_n(\beta+1)_n}{(n!)^2}\sum_{p,q=0}^{\infty}\frac{(-n)_{p+q}}{(\alpha+1)_p\,(\beta+1)_q}\frac{x^p\,y^q}{p!q!}\int_0^{\infty}e^{-\left(z+\frac{s}{2}\right)t}\,t^{b+p+q}\,W_{k,\mu}(st)\,dt \qquad \text{...... (2.3)}$$

By using (2.2), we get

$$A=\frac{(\alpha+1)_n(\beta+1)_n}{(n!)^2}\,\frac{\Gamma\left(b+\mu+\frac{3}{2}\right)\Gamma\left(b-\mu+\frac{3}{2}\right)s^{\mu+\frac{1}{2}}}{\Gamma(b-k+2)(z+s)^{b+\mu+\frac{3}{2}}}$$

$$\cdot\sum_{p,q=0}^{\infty}\frac{\left(b+\mu+\frac{3}{2}\right)_{p+q}\left(b-\mu+\frac{3}{2}\right)_{p+q}(-n)_{p+q}\left(\frac{x}{z+s}\right)^p\left(\frac{y}{z+s}\right)^q}{(b-k+2)_{p+q}\,(\alpha+1)_p\,(\beta+1)_q\,p!\,q!}$$

$$\times\,{}_2F_1\left[\begin{matrix}b+\mu+\frac{3}{2}+p+q,\ \mu-k+\frac{1}{2}; & \dfrac{z}{z+s}\\ b-k+2+p+q & ; \end{matrix}\right]. \qquad \text{...... (2.4)}$$

Now, by expanding ${}_2F_1$ into power series and adjusting the parameters, the result (2.4) yields the right-hand side of (2.1), and thereby (2.1) is proved.

Now, we consider the following two generating functions for Laguerre polynomial of two variable $L_n^{(\alpha,\beta)}(x,y)$ given by Chateerjea [1] and Khan and Shukla [4]

$$\sum_{n=0}^{\infty}\frac{n!L_n^{(\alpha,\beta)}(x,y)t^n}{(\alpha+1)_n(\beta+1)_n}=e^t\,{}_0F_1(-;\alpha+1;-xt)\,{}_0F_1(-;\beta+1;-yt) \qquad \text{...... (2.5)}$$

$$\sum_{n=0}^{\infty}\frac{n!(\lambda)_n L_n^{(\alpha,\beta)}(x,y)t^n}{(\alpha+1)_n(\beta+1)_n}=(1-t)^{-\lambda}\,\Psi_2\left[\lambda;\alpha+1,\ \beta+1;\frac{xt}{t-1},\frac{yt}{t-1}\right]. \qquad \text{... ... (2.6)}$$

In (2.5), if we replace x by xu and y by yu, multiply both the sides by $e^{-\left(z+\frac{s}{2}\right)u}\, u^{b} W_{k,\mu}(su)$ then integrate with respect to u between the limits 0 to ∞ by using the results (2.1) adjusting the parameters and by putting $b+\mu+\frac{3}{2}=A,\ b-\mu+\frac{3}{2}=B,\ b-k+2=C$ and s = 1, we get

$$\sum_{n=0}^{\infty} F^{(3)}\left[\begin{matrix} A::B,-n\ ;-;-:-;-;C-B; \\ C::-\ ;-;-:\alpha+1;\beta+1;\ -\ ; \end{matrix} \frac{x}{z+1},\frac{y}{z+1},\frac{z}{z+1}\right]\frac{t^{n}}{n!}$$

$$= F^{(3)}\left[\begin{matrix} A::B\ ;-;-:-;-;C-B; \\ C::-\ ;-;-:\alpha+1;\beta+1;\ -; \end{matrix} \frac{-xt}{z+1},\frac{-yt}{z+1},\frac{z}{z+1}\right] \qquad \text{...... (2.7)}$$

We adopt the same analysis that is employed to obtain (2.7) and, we use (2.1) to (2.6), following generating function is obtained

$$\sum_{n=0}^{\infty} F^{(3)}\left[\begin{matrix} A::B,-n\ ;-;-:-;-;C-B; \\ C::-\ ;-;-:\alpha+1;\beta+1;\ -\ ; \end{matrix} \frac{x}{z+1},\frac{y}{z+1},\frac{z}{z+1}\right]\frac{(\lambda)_n t^{n}}{n!}$$

$$= F^{(3)}\left[\begin{matrix} A::B,\lambda\ ;-;-:-;-;C-B; \\ C::-\ ;-;-:\alpha+1;\beta+1;-; \end{matrix} \frac{xt}{(z+1)(t-1)},\frac{yt}{(z+1)(t-1)},\frac{z}{z+1}\right]. \qquad \text{... ... (2.8)}$$

PARTICULAR CASES :

Now we mention some interesting special cases of the equations (2.7) and (2.8)

I- For B = C, (2.7) reduces to

$$\sum_{n=0}^{\infty} F_4\left[A,-n;\alpha+1,\beta+1;\frac{x}{z+1},\frac{y}{z+1}\right]\frac{t^n}{n!}$$

$$=e^{t}\,\Psi_2\left[A;\alpha+1,\beta+1;\frac{-xt}{z+1},\frac{-yt}{z+1}\right]. \qquad \text{....... (3.1)}$$

II- Letting x= 0 in (2.7) and taking B = 1+ β, we get

$$\sum_{n=0}^{\infty} F_1\left[A,-n,C-\beta-1;C;\frac{x}{z+1},\frac{y}{z+1}\right]\frac{t^n}{n!}$$

$$= e^{t}\, \Phi_1\left[A \;, C - \beta - 1 \;; C \;; \frac{z}{z+1}, \frac{-yt}{z+1} \right] \quad \ldots\ \ldots (3.2)$$

III- For B = C, (2.8) reduces to

$$\sum_{n=0}^{\infty} F_4\left[A\,, -n\,; \alpha+1\,, \beta+1\,; \frac{x}{z+1}, \frac{y}{z+1} \right] \frac{(\lambda)_n\, t^n}{n!}$$

$$= (1-t)^{-\lambda}\, F_4\left[A\,, \lambda\;; \alpha+1\,, \beta+1\;; \frac{xt}{(z+1)(t-1)}, \frac{yt}{(z+1)(t-1)} \right]. \quad \ldots\ldots (3.3)$$

VI- Letting x= 0 in (2.8) and taking B = 1+ β, we get

$$\sum_{n=0}^{\infty} F_1\left[A, -n, C-\beta-1; C; \frac{y}{z+1}, \frac{z}{z+1} \right] \frac{(\lambda)_n t^n}{n!}$$

$$= (1-t)^{-\lambda} F_1\left[A\;, \lambda\,, C-\beta-1\;; C\,;\; \frac{yt}{(t-1)(z+1)}, \frac{z}{z+1} \right], \quad \ldots\ \ldots (3.4)$$

where F_1 and F_4 are Appell's functions {[7], p.53 } and Ψ_2 *and* Φ_1 are Humbert's functions {[7]; p.59 }.

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GENERATING FUNCTIONS FOR THE GENERAL TRIPLE HYPERGEOMETRIC SERIES $F^{(3)}[x, y, z]$

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فـي هـذه الورقـة العلميـة أوجـدنا (generating functions) للدالـة $F^{(3)}(x, y, z)$ وذلـك باسـتخدام أسـلوب (integral operator) و بمـساعدة العلاقـتين (2.5) و (2.6). ففـي الجـزء الأول مـن هـذه الورقـة أعطينـا بعـض التعاريف لكل من الدوال التالية :

- Laguerre polynomials of two variables $L_n^{(\alpha,\beta)}(x, y)$.

- The general triple hypergeometric series $F^{(3)}\left[x,y,z\right]$.

أما في الجزء الثاني أوجدنا النتائج الرئيسية التالية :

$$1-\sum_{n=0}^{\infty} F^{(3)}\left[\begin{matrix} A::B,-n\;;-;-:-;-;C-B; \\ C::-\;;-;-:\alpha+1;\beta+1;\;-\;;\end{matrix} \frac{x}{z+1}, \frac{y}{z+1}, \frac{z}{z+1}\right]\frac{t^n}{n!}$$

$$= F^{(3)}\left[\begin{matrix} A::B\;;-;-:-;-;C-B; \\ C::-\;;-;-:\alpha+1;\beta+1;\;-;\end{matrix} \frac{-xt}{z+1}, \frac{-yt}{z+1}, \frac{z}{z+1}\right]$$

and

$$2-\sum_{n=0}^{\infty} F^{(3)}\left[\begin{matrix} A::B,-n\;;-;-:-;-;C-B; \\ C::-\;;-;-:\alpha+1;\beta+1;\;-\;;\end{matrix} \frac{x}{z+1}, \frac{y}{z+1}, \frac{z}{z+1}\right]\frac{(\lambda)_n t^n}{n!}$$

$$= F^{(3)}\left[\begin{matrix} A::B,\lambda\;;-;-:-;-;C-B; \\ C::-\;;-;-:\alpha+1;\beta+1;-;\end{matrix} \frac{xt}{(z+1)\,(t-1)}, \frac{yt}{(z+1)(t-1)}, \frac{z}{z+1}\right]$$

وفي الجزء الثالث اشرنا إلى بعض الحالات الخاصة (3.1) - (3.4) والمستنتجة من العلاقات الرئيسية (2.7) و (2.8).

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ON THE STRUCTURE OF SOME GROUPS OF DEGREE 11K CONTAINING M_{11}

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(Received: 24 April 2007)

Abstract: In this paper, we will show the structure of some groups containing the Mathieu group M_{11}, and generated by the two 5-cycles(k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)and the 11k-cycle (1, 2, ..., 11k) of degree 11k for all $2 \le k$. The structure of the groups founded is determined in terms of wreath product. Some related cases are also included.

1. Introduction The structure of all groups generated by an *n*-cycle and a 2-cycle or a 3-cycle or a product of two 2-cycles or a 4-cycle are determined by AL-Amri (see [1] and [2]). AL-Amri also determined the structure of groups generated by the *n*-cycle (1,2, ...,*n*) and a 5-cycle of the form $(a_1, a_2, a_3, a_4, a_5)$ for all $1 \le a_i \le n$, $a_i \ne a_j$ for all $1 \le i, j \le 5$ (see [3]).

The Mathieu group M_{11} of order 7920 is one of the well known simple groups. In [5], M_{11} is finitely presented as follows;

$$M_{11} = \langle A, B, C \mid A^{11} = B^5 = C^4 = (AC)^3 = 1, A^B = A^4, B^C = B^2 \rangle$$

The Mathieu group M_{11} is Also, generated using two permutations, the first is of order 11 and the second of order 5 as follows;

$$M_{11} = \langle (1, 2, ..., 11), (1, 2, 3, 7, 6)(4, 8, 5, 9, 10) \rangle.$$

In this paper, we will show the structure of the group generated by two permutations, the first is of order 11*k* and the second of order 5. We will show that the group obtained is the wreath product of M_{11} by C_k. Some related cases are also determined.

Keywords: group generated by *n*-cycle, wreath product of groups.

2. Preliminary Results

Definition 2.1 Let A and B be groups of permutations on non empty sets Ω_1 and Ω_2 respectively, where $\Omega_1 \cap \Omega_2 = \phi$. The wreath product of A and B is denote by A **wr** B

and defined as $A \mathbf{wr} B = A^{\Omega_2} \times_\theta B$, i.e., the direct product of $|\Omega_2|$ copies of A and a mapping θ where $\theta : B \to \mathrm{Aut}(A^{\Omega_2})$, is defined by $\theta_y(x) = x^y$, for all $x \in A^{\Omega_2}$. It follows that $|A \mathbf{wr} B| = (|A|)^{|\Omega_2|}|B|$.

Theorem 2.2 Let G be the group generated by the n-cycle $(1, 2, \ldots, n)$ and the 2-cycle (n, a). If $1 < a < n$ is an integer with $n = am$, then $G \cong S_m \mathbf{wr}\, C_a$.

3. The Results

Theorem 3.1 Let G be a group generated by the $11k$-cycle $(1, 2, \ldots, 11k)$ and the two 5-cycles$(k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)$. If $k > 1$ is an integer, then $G \cong M_{11} \mathrm{wr}\, C_k$ of order $(|M_{11}|)^k \times k$, where M_{11} is the Mathieu group of order 7920.

Proof : Let $\sigma = (1, 2, \ldots, 11k)$ and $\tau = (k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)$. Conjugating τ by σ^k we get $\eta_1 = (2k, 3k, 4k, 8k, 7k)(5k, 9k, 6k, 10k, 11k)$. Conjugating τ by σ^{2k} we get $\eta_2 = (3k, 4k, 5k, 9k, 8k)(6k, 10k, 7k, 11k, k)$. Hence $\delta = (\alpha_2^{-1})^{\alpha_1} = (k, 2k, 3k, 4k, 5k, 6k, 7k, 8k, 9k, 10k, 11k)$. Let $G_i = \langle \delta^{\sigma^i}, \tau^{\sigma^i} \rangle$ for all $1 \le i \le k$, be the groups acts on the sets $\Gamma_i = \{ k^{\sigma^i}, (2k)^{\sigma^i}, (3k)^{\sigma^i}, (4k)^{\sigma^i}, (5k)^{\sigma^i}, (6k)^{\sigma^i}, (7k)^{\sigma^i}, (8k)^{\sigma^i}, (9k)^{\sigma^i}, (10k)^{\sigma^i}, (11k)^{\sigma^i} \}$, for all $i = 1, 2, \ldots, k$, respectively. Since $\bigcap_{i=1}^{k} \Gamma_i = \phi$, then we get the direct product $G_1 \times G_2 \times \ldots \times G_k$, where each $G_i \cong M_{11}$. Let $\beta = \delta^{-1}\sigma = (1, 2, \ldots, k)(k+1, k+2, \ldots, 2k)\ldots(10k+1, 10k+2, \ldots, 11k)$. Let $H = \langle \beta \rangle \cong C_k$. H conjugates G_1 into G_2, G_2 into G_3, ... and G_k into G_1. Hence we get the wreath product $M_{11} \mathrm{wr}\, C_k \subseteq G$. On the other hand, since $\delta\beta = (1, 2, \ldots, k, k+1, k+2, \ldots, 2k, \ldots, 10k+1, 10k+2, \ldots, 11k) = \sigma$, then $\sigma \in M_{11} \mathrm{wr}\, C_k$. Hence $G = \langle \sigma, \tau \rangle \cong M_{11} \mathrm{wr}\, C_k$. ◊

Remark: Since

$$\delta = (k, 2k, 3k, 4k, 5k, 6k, 7k, 8k, 9k, 10k, 11k),$$

$$\beta = (1, 2, \ldots, k)(k+1, k+2, \ldots, 2k)\ldots(10k+1, 10k+2, \ldots, 11k) \text{ and}$$

$$\tau = (k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)$$

are in the group G described above, and since $\langle \delta, \tau \rangle \cong M_{11}$ then M_{11} wr $C_k \cong \langle \delta, \beta, \tau \rangle$. Hence M_{11} wr C_k can be finitely presented as follows

$$M_{11} \text{ wr } C_k \cong \langle X, Y, T \mid \langle X, Y \rangle \cong M_{11}, T^k = 1 \,.\, (XT)^{11k} = (YT)^{5k} = 1 \,\rangle.$$

Theorem 3.2 Let G be a group generated by the $11k$-cycle $(1, 2, \ldots, 11k)$, the two 5-cycles$(k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)$ and the involution $(1, 2)(k+1, k+2)(2k+1, 2k+2)\ldots(10k+1, 10k+2)$. If $k > 2$ is an integer, then $G \cong M_{11}$ wr S_k of order $(|M_{11}|)^k \times k!$.

Proof : Let $\sigma = (1, 2, \ldots, 11k)$, $\tau = (k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)$ and $\mu = (1, 2)(k+1, k+2)(2k+1, 2k+2)\ldots(10k+1, 10k+2)$. As in the proof of the previous theorem, $\langle \sigma, \tau \rangle \cong M_{11}$ wr C_k. Since $C_k = \langle (1,2,\ldots, k)(k+1,k+2,\ldots, 2k)\ldots(10k+1,10k+2,\ldots,11k) \rangle$,then $\langle\langle (1,2,\ldots, k)(k+1,k+2,\ldots, 2k)\ldots(10k+1,10k+2,\ldots,11k), \mu \rangle \cong S_k$. Hence $G = \langle \sigma, \tau, \mu \rangle \cong M_{11}$ wr S_k. ◊

As a consequence of the previous theorem we have the following result:

Theorem 3.3 Let G be a group generated by the $11k$-cycle $(1, 2, \ldots, 11k)$, the two 5-cycles$(k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)$ and the product of 3- cycles $(1, 2, 3)(k+1, k+2, k+3)(2k+1, 2k+2, 2k+3)\ldots(10k+1, 10k+2, 10k+3)$. If $k > 3$ is an odd integer, then $G \cong M_{11}$ wr A_k of order $(|M_{11}|)^k \times \frac{k!}{2}$.

Proof : The proof is similar to the proof of the previous result. ◊

THEOREM 3.4 Let $k = am \geq 4$ be any integer. G be a group generated by the $11k$-cycle $(1, 2, \ldots, 11k)$, the two 5-cycles$(k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k$ and the involution $(k, a)(2k, k+a)(3k, 2k+a)\ldots(11k, 10k+a)$, then $G \cong M_{11}$ wr $(S_m$ wr $C_a)$ of order $(|M_{11}|)^k \times (m!)^a \times a$.

Proof : Let $\sigma = (1, 2, \ldots, 11k)$, $\tau = (k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k)$ and $\mu = (k, a)(2k, k+a)(3k, 2k+a)\ldots(11k, 10k+a)$. As in the proof of Theorem 3.2, $\langle \sigma, \tau \rangle \cong M_{11}$ wr C_k. Since $C_k = \langle (1, 2, \ldots, k)(k+1, k+2, \ldots, 2k)\ldots(10k+1, 10k+2, \ldots, 11k) \rangle$, then $\langle (1, 2, \ldots, k)(k+1, k+2, \ldots, 2k)\ldots(10k+1, 10k+2, \ldots, 11k), \mu \rangle \cong (S_m$ wr $C_a)$. Hence $G = \langle \sigma, \tau, \mu \rangle \cong M_{11}$ wr $(S_m$ wr $C_a)$. ◊

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دراسة بناء بعض الزمر من الدرجة $11k$ والتى تحتوى زمرة ماثيو M_{11}

إبراهيم بن رشيد العمرى

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فى هذا البحث سوف نوضح بنية الزمرة من الدرجة $11k$ والتى تحتوى زمرة ماثيو من الدرجة M_{11} وتكون مولدة بعنصرين الأول عبارة عن حاصل ضرب دوارتين من النوع 5 والثانى عبارة عن دوارة من النوع $11k$ على النحو التالى: (k, 2k, 3k, 7k, 6k)(4k, 8k, 5k, 9k, 10k) و (1, 2, ... 11k). سوف نوضح تركيب الزمر الناتجة فى ضوء الضرب الريثى. كما أننا سوف نثبت بعض النتائج ذات العلاقة.