Delta J. Sci. 2008, 32: 1- 20 **MATHEMATICS**

Reducing four-level two-mode Hamiltonian to an effective two-level Hamiltonian with the addition of Kerr-like medium

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I. **Abstract:** We consider a two-mode quantized field described in a coherent state interacting with a four-level atom. An effective Hamiltonian is obtained by adiabatically eliminating the intermediate two levels in a cascade process. The influence of the Stark shifts and the Kerr-like medium on the atomic inversion are examined, as well as on the field entropy, atomic purity and Mandel's Q-parameter. The results of the calculations are illustrated numerically

1. Introduction:

The Jaynes-Cumming model (JCM) [1] has been recognized as the simplest and most effective model of the interaction between radiation and matter in quantum optics. It describes a two-level atom interacting with a single mode radiation in the rotating wave approximation (RWA). The model has been realized theoretically [2] and experimentally [3].The success of the JCM stimulates many physicists to extend and generalize the model in different ways. They study multimode and multiphoton instead of single mode and single photon [4], addition of Kerr-like medium and Stark shift [5] have been performed. Also, extensive studies of a three-level atom with different configurations under RWA interacting with quantized fields were carried out [6, 7].

Many authors have studied the entropy and the entanglement of the non-linear JCM, in different regime (see for example [8]). On the other hand, much attention has been focused on the properties of the entanglement between the field and the atom. The von-Neumann entropy has been proved to be a very useful operational measure of the purity of the quantum state [9]. The time evolution of the field (atomic) entropy reflects the time evolution of the degree of the entanglement between the atom and the field; the higher the entropy, the greater the entanglement. The field entropy for the entangled state of a single two-level atom interacting with a single field mode has been studied [10].The effect of the dynamic Stark shift on the evolution of the field entropy in the presence of a Kerr-like medium, for a single field mode, has been examined [11]. Studies for the field of quantum information and computing stimulates further investigation. The authors [12] extended the work to the case of two field modes.

In the present paper, we consider a generalization of the JCM and investigate the effect of Stark shift and Kerr-like medium on the atomic inversion, on the evolution of the field entropy and on the atomic purity. Moreover, we investigate the cavity field statistics through Mandel's Q parameter.

In the next section we consider the equations of motion in the Heisenberg picture for the system of two modes of the electromagnetic field interacting with a four-level atom in addition of Kerr-like medium. We derive the effective two-level Hamiltonian of the system which includes two-photon processes and the Stark-shift. The evolution operator and the wave function are obtained in section 3. Section 4 is devoted to the numerical investigations of the atomic inversion, the field entropy, atomic purity and Mandel Q-parameter. Finally, a conclusion is presented in section 5

2-The Basic Equations:

We consider a system of a four-level atom with non-equidistant, non-degenerate levels $|1\rangle, |2\rangle, |3\rangle and |4\rangle$ with energy levels Ω_k, (k=1, 2, 3, 4) respectively, interacting with two different modes of the electromagnetic field with frequencies ω_j, (j=1, 2).

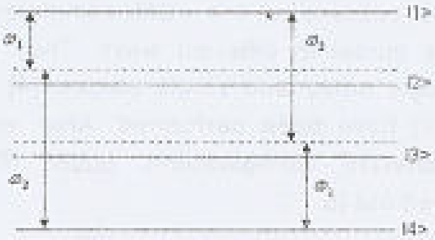


Fig.1

The Hamiltonian that describes the present system sketched in Fig. (1), within the rotating wave approximation has the following form (with $\hbar = 1$)

$$H=\sum_{j=1}^{2}\omega_j a_j^{+}a_j+\sum_{j=1}^{2}\chi_j a_j^{+2}a_j^2+\sum_{k=1}^{4}\Omega_k S_{kk}+\lambda_1(a_1^{+}S_{21}+hc)+\lambda_2(a_1^{+}S_{43}+h.c.)$$

$$+\lambda_3(a_2^{+}S_{31}+h.c)+\lambda_4(a_2^{+}S_{42}+h.c) \tag{1}$$

here $a_j(a_j^{+})$ is the annihilation (creation) operator for the j th mode of the field and satisfy the Boson commutation relation $\left[a_i,a_j^{+}\right]=\delta_{ij}$. $\lambda_k;(k=1,2,3,4)$ are the coupling constants and χ_j, j=1,2 describe the dispersive part of the third-order nonlinearity of the Kerr-like medium. The atomic operators S_{lk} satisfy the commutation relation [13] $\left[S_{ij},S_{lk}\right]=S_{ik}\delta_{lj}-S_{lj}\delta_{ik}$.

In order to apply the adiabatic elimination method to eliminate the two levels $|2\rangle$ *and* $|3\rangle$ we need to introduce the slowly varying operators [14]

$$\begin{aligned}&A_j=a_j e^{i\omega_j t},Q_{lk}=S_{lk}e^{-i\Omega_{lk}t};(l,k=1,2,3,4),(j=1,2)\\&where\ \Omega_{lk}=\Omega_l-\Omega_k\end{aligned} \tag{2}$$

The Heisenberg equation of motion for any operator O is

$$i\frac{\partial O}{\partial t}=[O,H]$$

Therefore, we have the following equations of motion for the atomic operators

$$\begin{aligned}
i\frac{dQ_{11}}{dt}&=-\lambda_1(A_1^{+}Q_{21}e^{-i\Delta t}-h.c)-\lambda_3(A_2^{+}Q_{31}e^{-i\Delta t}-h.c)\\
i\frac{dQ_{22}}{dt}&=\lambda_1(A_1^{+}Q_{21}e^{-i\Delta t}-h.c)-\lambda_4(A_2^{+}Q_{42}e^{i\Delta t}-h.c)\\
i\frac{dQ_{33}}{dt}&=\lambda_3(A_2^{+}Q_{31}e^{-i\Delta t}-h.c)-\lambda_2(A_1^{+}Q_{43}e^{i\Delta t}-h.c)\\
i\frac{dQ_{44}}{dt}&=\lambda_4(A_2^{+}Q_{43}e^{i\Delta t}-h.c)+\lambda_2(A_1^{+}Q_{43}e^{i\Delta t}-h.c)
\end{aligned} \tag{3}$$

$$i\frac{dQ_{12}}{dt} = \lambda_1 A_1^{+}(Q_{11} - Q_{22})e^{-i\Delta t} - \lambda_3 A_2^{+} Q_{32} e^{-i\Delta t} + \lambda_4 A_2 Q_{14} e^{-i\Delta t}$$

$$i\frac{dQ_{24}}{dt} = \lambda_4 A_2^{+}(Q_{22} - Q_{44})e^{i\Delta t} + \lambda_2 A_1^{+} Q_{23} e^{i\Delta t} - \lambda_1 A_1 Q_{14} e^{i\Delta t}$$

$$i\frac{dQ_{13}}{dt} = \lambda_3 A_2^{+}(Q_{11} - Q_{33})e^{-i\Delta t} - \lambda_1 A_1^{+} Q_{23} e^{-i\Delta t} + \lambda_2 A_1 Q_{14} e^{-i\Delta t} \qquad (4)$$

$$i\frac{dQ_{34}}{dt} = \lambda_2 A_1^{+}(Q_{33} - Q_{44})e^{i\Delta t} - \lambda_4 A_2^{+} Q_{32} e^{i\Delta t} - \lambda_3 A_2 Q_{14} e^{i\Delta t}$$

$$i\frac{dQ_{14}}{dt} = -\lambda_1 A_1^{+} Q_{24} e^{-i\Delta t} + \lambda_4 A_2^{+} Q_{12} e^{i\Delta t} - \lambda_3 A_2^{+} Q_{34} e^{-i\Delta t} + \lambda_2 A_1^{+} Q_{13} e^{i\Delta t}$$

$$(5)$$

Similarly for the two modes of the field, we have the equations

$$i\frac{dA_1}{dt} = 2\chi_1 A_1^{+} A_1^{2} + \lambda_1 Q_{21} e^{-i\Delta t} + \lambda_2 Q_{43} e^{i\Delta t} \qquad (6)$$

$$i\frac{dA_2}{dt} = 2\chi_2 A_2^{+} A_2^{2} + \lambda_3 Q_{13} e^{-i\Delta t} + \lambda_4 Q_{42} e^{i\Delta t} \qquad (7)$$

Where the detuning parameter Δ is defined as

$$\Delta = \Omega_{12} - \omega_1 = \omega_1 - \Omega_{34}$$
$$= \Omega_{13} - \omega_2 = \omega_2 - \Omega_{24} \qquad (8)$$

In order to integrate equation (4), we perform the slowly varying amplitudes approximation for both the atomic amplitudes and for the field envelopes. So we replace Q_{ij}(t) and A_j (t) inside the integral with their values at the upper limit of the integration and carrying out the integration on the remaining exponential factors from 0 to t (with initial condition Q_{ij} (0) = 0, $i \neq j$).Thus we get the following eqns.

$$Q_{12} = \frac{1}{\Delta}\left[\lambda_1 A_1^{+}(Q_{11} - Q_{22}) + \lambda_4 A_2 Q_{14} - \lambda_3 A_2^{+} Q_{32}\right]e^{-i\Delta t}$$

$$Q_{24} = \frac{-1}{\Delta}\left[\lambda_4 A_2^+ (Q_{22} - Q_{44}) - \lambda_1 A_1 Q_{14} + \lambda_2 A_1^+ Q_{23}\right] e^{i\Delta t}$$

$$Q_{13} = \frac{1}{\Delta}\left[\lambda_3 A_2^+ (Q_{11} - Q_{33}) + \lambda_4 A_1 Q_{14} - \lambda_1 A_1^+ Q_{23}\right] e^{-i\Delta t} \tag{9}$$

$$Q_{34} = \frac{-1}{\Delta}\left[\lambda_2 A_1^+ (Q_{33} - Q_{44}) - \lambda_3 A_2 Q_{14} + \lambda_4 A_2^+ Q_{32}\right] e^{i\Delta t},$$

By substituting eqns. (9) into (3) we get

$$Q_{22} + Q_{33} = Const. \quad \text{i.e.,} \; S_{22} + S_{33} = Const. \tag{10}$$

which means that the occupation of levels $|2\rangle and |3\rangle$ is constant during the interaction.

With help of eqns. (9) we can, directly, obtain from eqns. (5-7) the following equations of motion

$$i\frac{dS_{14}}{dt} = \Omega_{41} S_{14} - \lambda a_1^+ a_2^+ (S_{44} - S_{11}) - \left[\frac{1}{\Delta}(\lambda_1^2 - \lambda_2^2) a_1^+ a_1 + \frac{1}{\Delta}(\lambda_3^2 - \lambda_4^2) a_2^+ a_2\right] S_{14} \tag{11}$$

$$i\frac{da_1}{dt} = \omega_1 a_1 + 2\chi_1 a_1^+ a_1^2 + \frac{1}{\Delta}(\lambda_3^2 S_{11} + \lambda_2^2 S_{44}) a_1 + \lambda a_2^+ S_{41} \tag{12}$$

$$i\frac{da_2}{dt} = \omega_2 a_2 + 2\chi_2 a_2^+ a_2^2 + \frac{1}{\Delta}(\lambda_3^2 S_{11} + \lambda_4^2 S_{44}) a_2 + \lambda a_1^+ S_{41} \tag{13}$$

where,

$$\lambda = \frac{1}{\Delta}(\lambda_1 \lambda_4 + \lambda_2 \lambda_3) = \beta_1 + \beta_2 \tag{14}$$

The equations (11-13) can be considered as the equations of the motion according to the following Hamiltonian

$$H_{eff}=\sum_{j=1}^{2}\omega_j a_j^{+}a_j+\sum_{j=1}^{2}\chi_j a_j^{+2}a_j^{2}+\Omega_1 S_{11}+\Omega_4 S_{44}+a_1^{+}a_1\left(\frac{\lambda_1^2}{\Delta}S_{11}+\frac{\lambda_2^2}{\Delta}S_{44}\right)$$
$$+a_2^{+}a_2\left(\frac{\lambda_3^2}{\Delta}S_{11}+\frac{\lambda_4^2}{\Delta}S_{44}\right)+\lambda(a_1^{+}a_2^{+}S_{41}+S_{14}a_1a_2) \qquad (15)$$

Since we have only two effective levels $|1\rangle$ *and* $|4\rangle$ in the above Hamiltonian, we will replace $|4\rangle$ *by* $|2\rangle$ just for convenience and Heff can be rewritten as

$$H_{eff}=\sum_{j=1}^{2}\omega_j a_j^{+}a_j+\sum_{j=1}^{2}\chi_j a_j^{+}a_j(a_j^{+}a_j-1)+\sum_{k=1}^{2}\Omega_k S_{kk}+\frac{1}{\Delta}S_{11}(\lambda_1^2a_1^{+}a_1+\lambda_3^2a_2^{+}a_2)$$
$$+\frac{1}{\Delta}S_{22}(\lambda_2^2a_1^{+}a_1+\lambda_4^2a_2^{+}a_2)+\lambda(a_1^{+}a_2^{+}S_{21}+S_{12}a_1a_2) \qquad (16$$

In equation (16), we note that, the 4th and the 5th terms represent the Stark-shift [14] of the levels $|1\rangle$ *and* $|2\rangle$ due to the field modes j=1, 2, respectively. Note the difference in the form in which it appears here compared to the single mode of the three level cases [14,15]. We note that the shifts are affected by the two modes with different weights depending on the original coupling constants.

3.1 The Time Evolution Operator

The Heisenberg equations of motion for the operators

$n_j=a_j^{+}a_j$, j = 1, 2 and S_{11} are

$$i\frac{dn_1}{dt}=L \quad , \quad i\frac{dn_2}{dt}=L \quad \text{and} \quad i\frac{dS_{11}}{dt}=-L \qquad (17)$$

where ,

$$L=\lambda(a_1^{+}a_2^{+}S_{21}-S_{12}a_1a_2) \qquad (18)$$

We deduce that the following operators

$$n_1+S_{11}=N_1 \quad \text{and} \quad n_2+S_{11}=N_2 \qquad (19)$$

are constants of the motion.

Thus, the Hamiltonian (16) can be written as

$$H_{eff} = N + C + \gamma I \tag{20}$$

$$N = Z_1 S_{11} + Z_2 S_{22} \tag{21}$$

with

$$Z_1(n_1,n_2) = \omega_1(n_1+1) + \omega_2(n_2+1) + \frac{1}{2\Delta}\left[(\lambda_1^2+\lambda_2^2)(n_1+1) + (\lambda_3^2+\lambda_4^2)(n_2+1)\right] + \chi_1(n_1-1)(n_1+1) + \chi_2(n_2-1)(n_2+1) \tag{22}$$

$$Z_2(n_1,n_2) = \omega_1 n_1 + \omega_2 n_2 + \frac{1}{2\Delta}\left[(\lambda_1^2+\lambda_2^2)n_1 + (\lambda_3^2+\lambda_4^2)n_2\right] + \chi_1 n_1(n_1-2) + \chi_2 n_2(n_2-2) \tag{23}$$

$$\gamma = \frac{1}{2}(\Omega_1+\Omega_2-\omega_1-\omega_2) - \frac{1}{2}\left(\frac{\lambda_1^2}{\Delta}+\frac{\lambda_3^2}{\Delta}\right) + (\chi_1+\chi_2) \tag{24}$$

and I is the identity operator.

We note that

$$a_1 a_2 Z_2(n_1,n_2) = Z_1(n_1,n_2) a_1 a_2 \ ; \ Z_2(n_1,n_2) = Z_1(n_1-1,n_2-1) \tag{25}$$

and $\quad C = \Lambda + L$ (26)

with $\quad \Lambda = \delta_1 S_{11} - \delta_2 S_{22}$ (27)

where

$$\delta_1(n_1,n_2) = \delta + \frac{1}{2\Delta}\left[\lambda_1^2 n_1 - \lambda_2^2(n_1+1) + \lambda_3^2 n_2 - \lambda_4^2(n_2+1)\right] - \chi_1 n_1 - \chi_2 n_2 \tag{28}$$

$$\delta_2(n_1,n_2) = \delta + \frac{1}{2\Delta}\left[\lambda_1^2(n_1-1) - \lambda_2^2 n_1 + \lambda_3^2(n_2-1) - \lambda_4^2 n_2\right] - \chi_1(n_1-1) - \chi_2(n_2-1) \tag{29}$$

The detuning parameter δ is given by

$$\delta = (\Omega_1 - \Omega_2) - (\omega_1 + \omega_2) = 0 \tag{30}$$

where eqn.(8) is used

Also we note that

$$a_1 a_2 \delta_2(n_1,n_2) = \delta_1(n_1,n_2) a_1 a_2 \ ; \ \delta_2(n_1,n_2) = \delta_1(n_1-1,n_2-1) \tag{31}$$

It is easy to show that the operators N and C commute with each other and hence with H. This means that N and C are constants of motion.

The time evolution operators for our system is given by

U(t)= exp (-iγt) exp (-iNt) exp (-iCt) (32)

where

$$\exp(-iNt) = \begin{bmatrix} e^{-i\varepsilon_1 t} & 0 \\ 0 & e^{-i\varepsilon_2 t} \end{bmatrix} \tag{33}$$

$$\exp(-iCt) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{34}$$

with

$$A_{11} = Cos\eta_1 t - i\delta_1 \frac{Sin\eta_1 t}{\eta_1} \tag{35}$$

$$A_{12} = -i\lambda \frac{Sin\eta_1 t}{\eta_1} a_1 a_2 \tag{36}$$

$$A_{21} = -i\lambda a_1^+ a_2^+ \frac{Sin\eta_1 t}{\eta_1} \tag{37}$$

$$A_{22} = Cos\eta_2 t + i\delta_2 \frac{Sin\eta_2 t}{\eta_2} \tag{38}$$

where,

$$\eta_i^2(n_1,n_2) = \delta_i^2(n_1,n_2) + \nu_i(n_1,n_2)\ ,\ \ i = 1,2 \tag{39}$$

with

$$\nu_1(n_1,n_2) = \lambda^2(n_1+1)(n_2+1)\ ;\ \nu_2(n_1,n_2) = \nu_1(n_1-1,n_2-1) \tag{40}$$

Evaluating the time evolution operator enable us to discuss the dynamical behavior of the system.

3.2 The Wave Function

Let us consider that, at time t=0, the effective two-level atom is in a coherent atomic state

$$|\theta,\phi\rangle = Cos\frac{\theta}{2}|1\rangle + e^{-i\phi}Sin\frac{\theta}{2}|2\rangle \tag{41}$$

Where $|1\rangle$ and $|2\rangle$ stands for the excited and ground states of the atom respectively. ϕ is the relative phase of the two atomic levels. When $\theta \to 0$ the excited state is considered while when $\theta \to \pi$ then the wave function describes the atom in its ground state. Also we consider the fields to be initially in the uncorrelated coherent states $|\alpha_1,\alpha_2\rangle = |\alpha_1\rangle \otimes |\alpha_2\rangle$. Then the initial state of the fields takes the following form

$$|\alpha_1,\alpha_2\rangle = \sum_{n_1,n_2} q_{n_1,n_2}|n_1,n_2\rangle \tag{42}$$

where $q_{n_1,n_2} = q_{n_1}q_{n_2}, q_{n_i}$ describes the amplitude of the state $|n_i\rangle$ of the ith mode, with $q_{ni} = \exp(-\frac{1}{2}|\alpha_i|^2)\frac{\alpha_i^{n_i}}{\sqrt{n_i!}}$. At time t=0 the wave function $\psi(0)$ is given by $|\psi(0)\rangle = |\theta,\phi\rangle \otimes |\alpha_1,\alpha_2\rangle$.

At the time t > 0 the wave function takes the form

$$|\psi(t)\rangle = |S(t)\rangle|1\rangle + |T(t)\rangle|2\rangle \tag{43}$$

Where

$$|S(t)\rangle = e^{-i\varepsilon_1 t}\left[\cos\left(\frac{\theta}{2}\right)A_{11} + e^{i\phi}\sin\left(\frac{\theta}{2}\right)A_{12}\right]|\alpha_1,\alpha_2\rangle \tag{44}$$

and

$$|T(t)\rangle = e^{-i\varepsilon_2 t}\left[\cos\left(\frac{\theta}{2}\right)A_{21} + e^{i\phi}\sin\left(\frac{\theta}{2}\right)A_{22}\right]|\alpha_1,\alpha_2\rangle \tag{45}$$

with A_{ij} given by eqns. (35-38). For an atom in the excited state ($\theta \to 0$) we have, by using eqns. (35-38),

$$|S(t)\rangle = e^{-i\varepsilon_1 t}\sum_{n_1,n_2=0}^{\infty} e^{-\frac{1}{2}|\bar{\alpha}|^2}\frac{\alpha_1^{n_1}\alpha_2^{n_2}}{\sqrt{n_1!n_2!}}\left(\cos\eta_1 t - i\delta_1\frac{\sin\eta_1 t}{\eta_1}\right)|n_1,n_2\rangle \tag{46}$$

and

$$|T(t)\rangle = -i\lambda e^{-i\varepsilon_1 t}\sum_{n_1,n_2=0}^{\infty} e^{-\frac{1}{2}|\bar{\alpha}|^2}\frac{\alpha_1^{n_1}\alpha_2^{n_2}}{\sqrt{n_1!n_2!}}\sqrt{(n_1+1)(n_2+1)}\frac{\sin\eta_1 t}{\eta_1}|n_1+1,n_2+1\rangle \tag{47}$$

where $|\bar{\alpha}|^2 = |\alpha_1|^2 + |\alpha_2|^2$

Calculating the wave function enables us to investigate any phenomenon related to the atom and the field modes.

4. Discussion of atomic and field dynamics:

In this section, we investigate some aspects of our system, namely, the atomic inversion, the field entropy, the atomic purity and Mandel's Q-parameter.

4.1 The Atomic Inversion

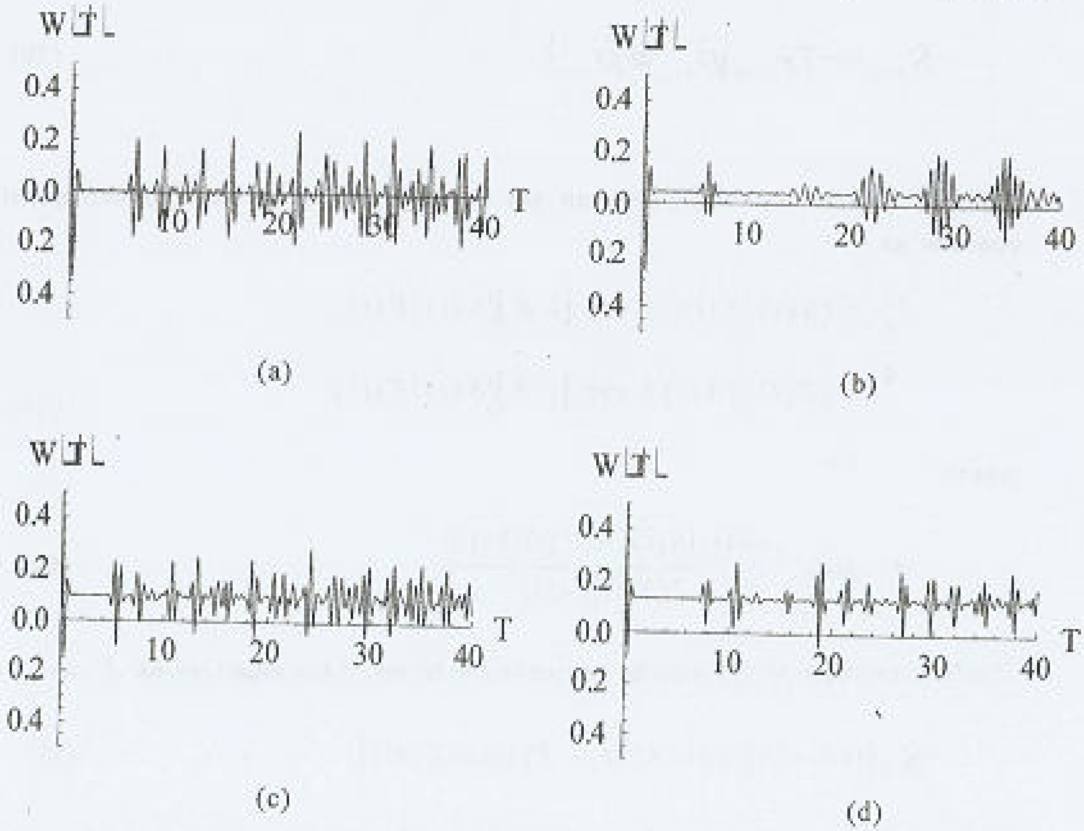
The atomic population inversion is defined as the difference between the probabilities of finding the atom in the excited state and in the ground state. When the atom starts in its excited state, the atomic inversion takes the form

$$W(t) = \frac{1}{2}[\langle S_{11}\rangle - \langle S_{22}\rangle]$$

$$= \frac{1}{2} - \sum_{n_1,n_2}^{\infty} e^{-|\bar{\alpha}|^2}\frac{|\alpha_1|^{2n_1}|\alpha_2|^{2n_2}}{n_1!n_2!}\nu_i\frac{Sin^2\eta_1 t}{\eta_1^2} \tag{48}$$

We plot the atomic population inversion W (T) against the scaled time T=λ t with the intensity of the initial coherent field equal to $\bar{n}_j = 4$, j=1,2 Fig.(2). First we ignore the terms (δ_i^2 in eq.(39)) which represent the Stark shift and Kerr-like medium, the result is illustrated in Fig.2(a).It is apparent that the oscillations are around W(T)=0.They start after the first collapse but become irregular as T increases. We know that the collapses are caused by the dephasing of the various teams in the sums in eq. (48).The effect of the Stark shift, only, on the atomic inversion appears clearly in Fig.2 (b) where the oscillations of the atomic inversion W (T) show the collapse and revival phenomenon. This phenomenon arises from the presence of the interference between the multiple exchanges of photons involved in exciting and de- exciting the atom from the cavity modes. The base line of W (T) in Fig.2 (b) is shifted upward which means more energy is stored in the atomic system.

To visualize the influence of the Kerr-like medium in the atomic inversion we set χ ($\chi_1 = \chi_2 = \chi$) at the value of 0.3 in Fig 2(c).We see that the atomic inversion oscillates about a positive value. The collapse intervals are shorter than that in Fig.2 (a). As the time T increases the behavior of irregular oscillation is more pronounced. However the atomic system has more energy, the amplitude of the oscillations are decreased. Fig2 (d) shows effects of both the Stark shift and the Kerr-like medium. The atomic system gains little more energy and the oscillations are decreasing as the time increase. Therefore the Stark shift and the Kerr-like medium inhibit the atomic system from giving its total energy to the field.



Fig(2) The atomic population inversion W(T) versus the scaled time T= λ t when the atom starts in the excited state and the field in coherent states with $\bar{n}_1 = \bar{n}_2 = 4$ The Stark shift parameters β1, β2 and the Kerr-like medium parameter χ are in(a) β1 = β2 =0.0, χ =0.0, (b) β1 = (0.995)2, β2 = (0.1)2, χ =0.0,

(c) β1 = β2 = 0.0, χ = 0.3, (d) β1 = (0.995)2, β2 = (0.1)2 , χ = 0.3

4-2 Entropy of the Field

We use the field entropy as a measurement of the degree of entanglement between the field and the atomic system. As it is known, the Von-Neumann entropy S of a quantum mechanical system is defined as

$$S = -Tr\left\{\hat{\rho}\ln\hat{\rho}\right\} \tag{49}$$

where $\hat{\rho}$ is the density operator for the system .We have set Boltzmann's

constant K=1. If $\hat{\rho}$ describes a pure state, then $S = 0$ while if $\hat{\rho}$ describes a mixed state, then $S \neq 0$. The sub-entropies of the field and atomic sub-system are defined by the corresponding reduced density operators [17]

$$S_{f(a)} = -Tr_{f(a)}\left\{\hat{\rho}_{f(a)}\ln\hat{\rho}_{f(a)}\right\} \tag{50}$$

Applying the method of [9] we can get the eigenvalues of the reduced density operator as

$$\begin{aligned}\lambda_f^{\pm} &= \langle S(t)|S(t)\rangle \pm \exp\left[\mp\vartheta\right]\langle S(t)|T(t)\rangle \\ &= \langle T(t)|T(t)\rangle \pm \exp\left[\pm\vartheta\right]\langle S(t)|T(t)\rangle\end{aligned} \tag{51}$$

where

$$\vartheta = shin^{-1}\frac{\langle S(t)|S(t)\rangle - \langle T(t)|T(t)\rangle}{2\langle S(t)|T(t)\rangle} \tag{52}$$

The field entropy $S_f(t)$ may be expressed in terms of the eigenvalues $\lambda_f^{\pm}$ as

$$S_f(t) = -\left[\lambda_f^{+}(t)\ln\lambda_f^{+}(t) + \lambda_f^{-}(t)\ln\lambda_f^{-}(t)\right] \tag{53}$$

As an initial condition we have taken the coherence parameter α_j to be real where $\alpha_j^2 = \bar{n}_j = 4,\ j = 1,2$ .An atom is assumed to be in its excited state, (θ=0 φ= 0). We display the evolution of the field entropy for the two-mode field interacting with the considered effective two-level atomic system against the scaled time T = λt. The outcome is presented in Fig (3). In all cases we notice that there are periodic changes in the field entropy and these are due to the existence of the periodic functions in the expression of $S_f(t)$. The evolution of the field entropy in the case of two-photon process is rather different compared with the one-photon case [9] where the oscillating period is longer than that of the two-photon case. The field entropy evolves to the minimum values when

T = (n+1/2) π, (n= 0, 1, 2, ...) and the field is partially disentangled from the atom while when T= n π the field entropy evolves to maximum values and the field is strongly entangled with the atom.

To visualize the influence of the Stark shift we compare Fig.3 (a) where the Stark effect is absent and (b) where the Stark shift parameters β1, β2 have equal values, we see that the evolution of the field entropy is almost similar for both cases. This is because the two effective levels of the atom are equally coupled with the intermediate levels. From Fig.3 (d) we notice that increasing the difference between Stark shift parameters leads to an increase in the minimum values of the entropy and the atom-field entanglement is reduced. The entropy fluctuations become smoother especially at the beginning of the interaction.

We display the evolution of the field entropy considering the effect of the Kerr-like medium in Fig.3(c, e) for different values of χ (where we set $\chi_1 = \chi_2 = \chi$).It is clear that increasing the nonlinear interaction of the Kerr-like medium with the field results in decreasing the entropy. In this case the degree of the entanglement between the field and the atom is reduced. The amplitude of the field entropy decreases as the value of χ increases and the oscillations become very condensed. Including both the effects of Stark shift and Kerr-like medium, Fig. 3 (f), leads to more reduction in the amplitude of the field entropy and in its fluctuations. Finally, in the strong nonlinear interaction of the Kerr-like medium with the field modes limit for χ > 1.0, numerical studies show that the entropy tends almost to zero and the field can be sustained in a pure state. This result corresponds to the fact that in a strong nonlinear interaction of the Kerr-like medium with the field modes, the field and the atom are almost decoupled and the time evolution of the field is governed by the Hamiltonian H ' eff = $\sum_{j=1}^{2} \chi_j a_j^{+2} a_j^2$.

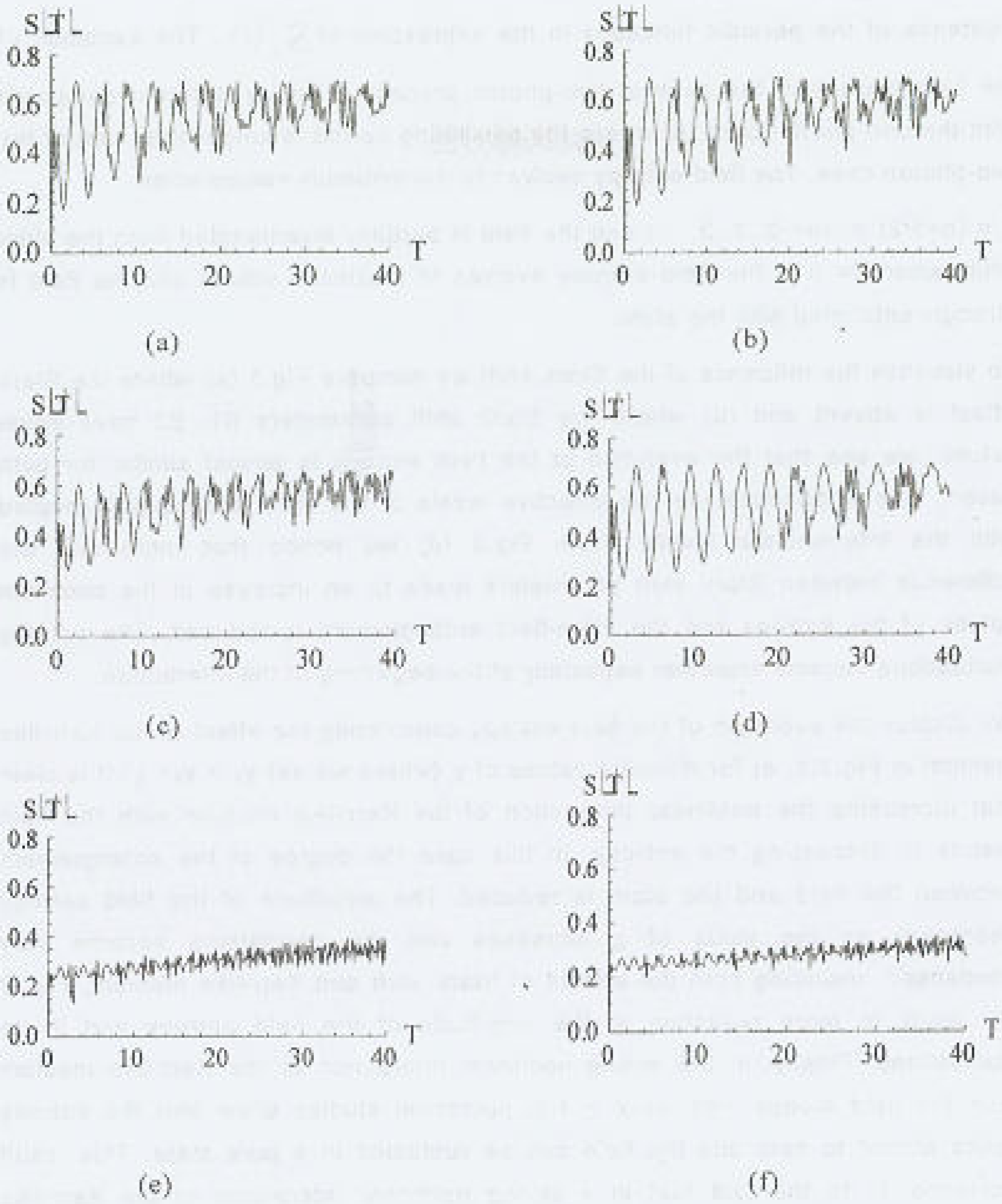


Fig.3 Evolution of the field entropy versus the scaled time $T = \lambda t$

(a) $\beta_1 = \beta_2 = 0.0$, $\chi = 0.0$, (b) $\beta_1 = \beta_2 = (0.71)^2$, $\chi = 0.0$,

(c) $\beta_1 = \beta_2 = 0.0$, $\chi = 0.3$, (d) $\beta_1 = (0.995)^2, \beta_2 = (0.1)^2$, $\chi = 0.0$,

(e) $\beta_1 = \beta_2 = 0.0$, $\chi = 1.0$, (f) $\beta_1 = (0.995)^2, \beta_2 = (0.1)^2$, $\chi = 1.0$

4.3 The purity

The purity of the atomic state can be determined by the time evolution of the quantity $Tr\left\{ \left(\rho^A(t)\right)^2 \right\}$. The purity is defined as [18]

$$P_u(t) = 1 - Tr\left\{ \left(\rho^A(t)\right)^2 \right\} \tag{54}$$

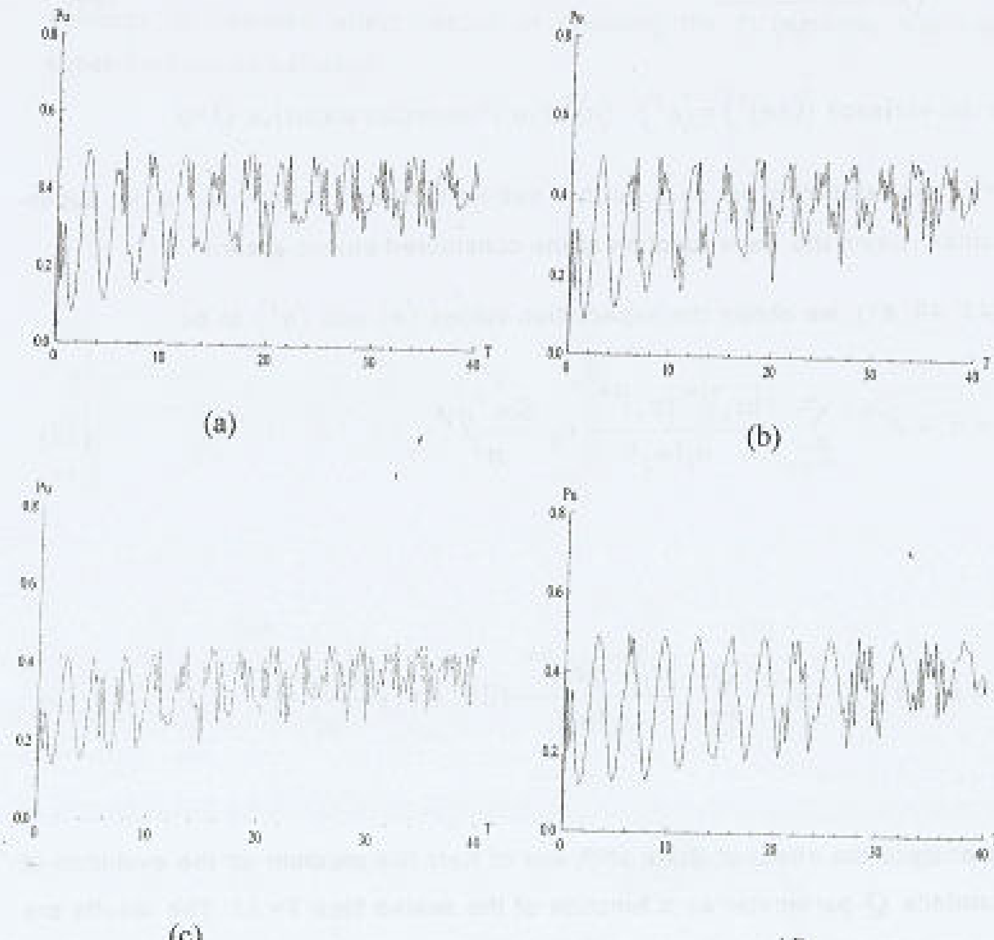
where $\rho^A(t)$ is given by

$$\rho^A(t) = \begin{pmatrix} \langle S(t)|S(t)\rangle & \langle S(t)|T(t)\rangle \\ \langle T(t)|S(t)\rangle & \langle T(t)|T(t)\rangle \end{pmatrix} \tag{55}$$

Therefore we can write the purity as follows

$$p_u(t) = 2\left[\langle S(t)|S(t)\rangle \langle T(t)|T(t)\rangle - \left| \langle S(t)|T(t)\rangle \right|^2 \right] \tag{56}$$

we display the evolution of the purity in Fig.4. The Stark shift and the Kerr-medium parameters have the same values as in Fig.3.We observe that the maximum and the minimum values of the purity decease as the effect of nonlinear Kerr-like medium increase (through the parameters χ_i, Fig.3 (a, c, e).Comparing Fig.3 (b, d), the influence of the Stark shift on the purity leads to a very small reduction. It is noted that neither the field nor the atom can become in pure state once the interaction is started.



(a) (b)

(c) (d)

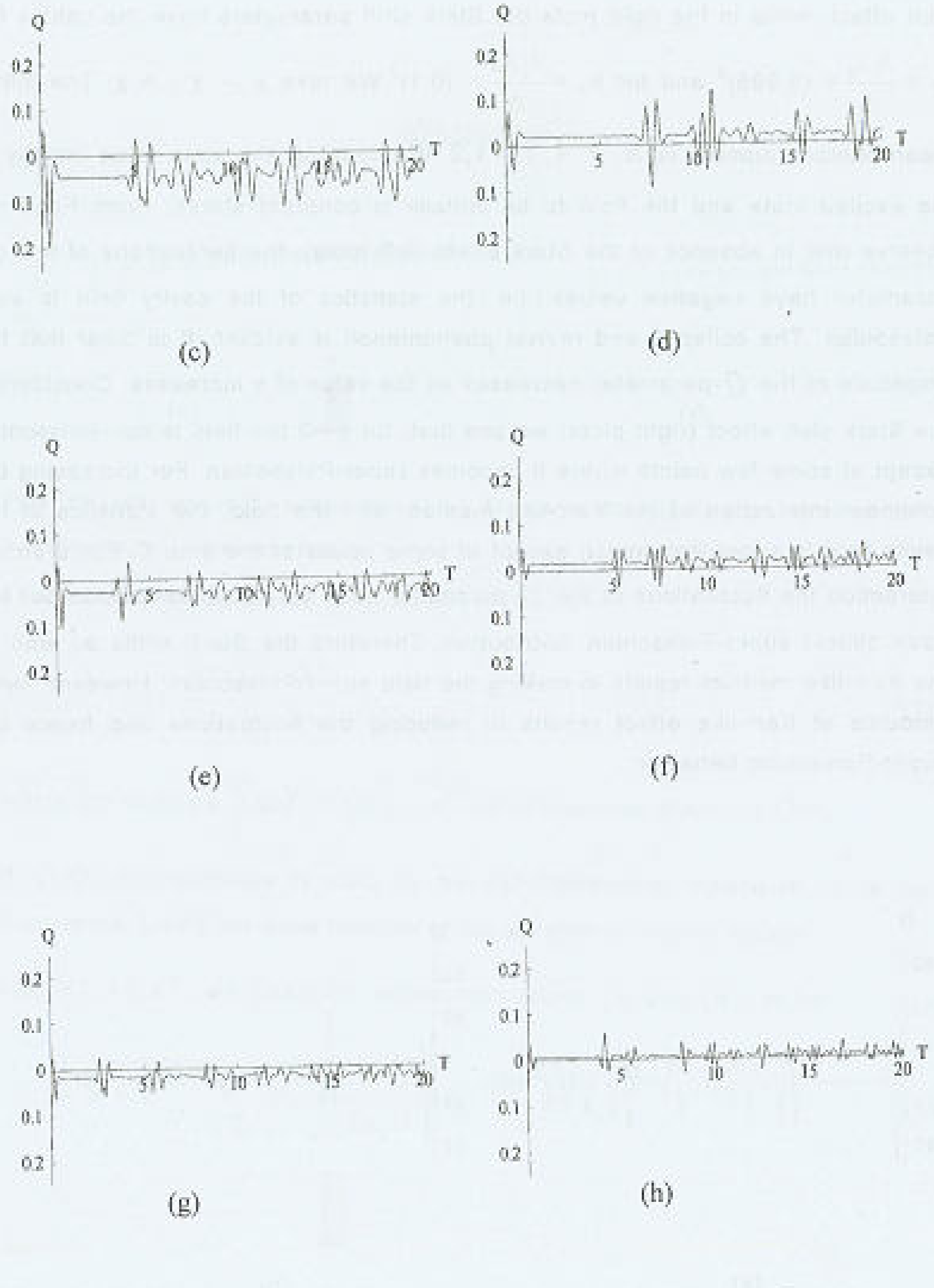


Fig.5
Evolution of Q-parameter versus the scaled time T= λ t when the atom starts in the excited state and the field in coherent states with $\bar{n}$ =4 .Left plots are for Stark shift parameters β_1 = β_2 = 0.0, right plots are for
β_1 = $(0.995)^2$, β_2 = $(0.1)^2$ and in (a, b) χ = 0.0, (c, d) χ = 0.3, (e, f) χ = 0.7,
(g, h) χ = 1.0

5. Conclusion

In this paper, we consider a four-level atomic system interacting with two modes of quantized field. Within the RWA approximation we reduce this model to an effective two-level Hamiltonian by applying the adiabatic elimination method in order to eliminate the two intermediate levels. We obtain a new type of Stark shift different than the one that appears in the study of three-level atom. Also, the Kerr-like medium is considered. We calculate the wave function of the system and hence, we get the atomic population inversion, the field entropy and the purity of the atomic state. The Mandel's Q-parameter is also examined. By a numerical study we display the effects both of the Stark shift and Kerr-like medium.

These effects lead to the collapse and revival phenomenon to be more pronounced in the atomic inversion and inhibit the atomic system from giving its total energy to the field. The field entropy shows a reduction when the difference between the Stark shift parameters increase and when strong nonlinear interaction with the Kerr-like medium take place. In these cases the atom-field entanglement reduces. For the purity of the atomic state we notice that, considering the effects of the Stark shift as well as the Kerr medium, neither the field nor the atom can become in pure state once the interaction is started. For the Q-parameter we find that the Stark shift as well as the Kerr-like medium results in making the field super-Poissonian. However, large amounts of Kerr-like effect results in reducing the fluctuations and hence the super-Poissonian behavior.

In the absence of the Stark shift effect, the cavity field shows sub-Poissonian distribution, regardless of the strength of the interaction with the Kerr medium, considering that the Stark effect makes the field super-Poissonian.

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اختزال نظام ذرى ذو اربع مستويات طاقه يمتص اويشع فوتتين الى نظام ذرى ذو مستويين فاعلين من الطاقه تحت تاثير وسط كبير.

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تم فى هذا البحث استخدام تمثيل هيزينبرج فى استنتاج معادلات الحركة للمتغيرات الذرية و متغيرات المجال لنظام يتكون من ذرة ذات اربع مستويات من الطاقه تمتص او تشع فوتتين حتى تنتقل من مستوى الطاقه السفلى الى العلوى او العكس و ذلك فى ظل وجود وسط كبير.

وقد استخدمنا طريقة الحذف الادياباتيكى حتى نحذف المستويين الاوسطين من الطاقه لنحصل على معادله هاملتون الخاصه بذلك النظام فقط. نتيجه لذلك نحصل على نوع جديد من تأثير ستارك و هو مختلف عن النوع الناتج من الذرة ذات ثلاث مستويات طاقه.

قمنا بدراسه تأثير وجود وسط كبير و كذلك تأثير ستارك على انقلاب الاسكان الذرى، الصفاء الذرى، مقياس المعلوماتيه للمجال و هو المعروف بمدى اهميته فى المعلومات الكميه والكمبيوتر الكمى و اخيرا احصائيات المجال من خلال قياس معامل ماندل.