Copyright @ by Tanta University, Egypt

Delta J. Sci. 2009, 33: 10- 15 MATHEMATICS

1. ON STRONGLY $F_{S\beta}$-IRRESOLUTE MAPPINGS

R. K. SARAF & R. K. POURANIK *

Department of Mathematics, Govt.K.N.M. Damoh(M.P.)470661, INDIA

Department of Mathematics, Dr. V. L.M. College Damoh(M.P.) 470661,INDIA

(Received: 7-9-2009)

ABSTRACT: The aim of this paper is to introduce a new class of mappings called strongly $F_{S\beta}$ - irresolute mapping. Its characteristic properties, examples and composition with other mappings are studied.

Keywords: F_S-open, F_β- open, fuzzy topological space, st- $F_{S\beta}$ irresolute mapping.
2000 Math subject classification: 54A40, 54C10, 54D10.

1. INTRODUCTION

In this paper, we introduce a new class of mapping called st- $F_{S\beta}$ - irresolute mapping. In section 2 some examples and characterizations of new mappings are examined. In section 3 the composition of st-$F_{S\beta}$ - irresolute mapping with other fuzzy mappings are studied.

The concept fuzzy has invaded almost all branches of mathematics, since the introduction of the concept of fuzzy sets by Zadeh [15]. The theory of fuzzy topological spaces was introduced and developed by Chang [4] and since then various notions in classical topology has been extended to fuzzy topological spaces. Our motivation in this paper is to define strongly $F_{s\beta}$- irresolute mappings and investigate its properties The newly defined class of mapping is stronger then M-fuzzy β -continuos mapping and is a generalization of strongly $F_{\alpha p}$- irresolute mapping.

Throughout this note, spaces, always mean fuzzy topological spaces and f : X → Y denotes a mapping of a space X into a space Y. Let A be a fuzzy subset of a space X. The closure and the interior of a fuzzy set A are denoted by Cl (A) and Int (A) respectively. The notation and terminologies not explained in this paper may be found in [9]. Definitions and results. which will be needed in this paper, are recalled here:

DEFINITION 1.1 Let A be a fuzzy subset in a space X

(a) A is called :

(i) Fuzzy α -open [3] (shortly F_α-open) iff $A \le Int(Cl(Int(A)))$.

(ii) Fuzzy semi open [1] (shortly Fs open) iff $A \le Cl(Int(A))$.

(iii) Fuzzy pre-open [3] (shortly Fp –open) iff $A \le Int(Cl(A))$.

(iv) Fuzzy β-open [5] (shortly F_β-open) iff $A \leq Cl(Int(Cl(A)))$.

The complement of these sets are respectively called F_α- closed (resp. Fs- closed, Fp- closed, F_β-closed).

(b) (i) The fuzzy pre- interior [12] (resp. fuzzy β-interior [2]) of A, denoted by pInt (A) (resp. βInt (A)) is the union of all F_p-open (resp. F_β-open) subsets contained in A.

(ii) The fuzzy pre closure[12] (resp. fuzzy β-closure [2]) of A, denoted by pCl (A) (resp. βCl (A)) is the intersection of all F_p- Closed (resp. F_β-closed) subsets containing A.

(c) Let $f : X \to Y$ be mapping. Then f is called :

(i) Strongly fuzzy α pre irresolute [11] (shortly st- $F_{\alpha p}$- irresolute) if $f^{-1}(A)$ is F_α-open in X, for every F_β-open set A of Y.

(ii) M-fuzzy β-continuous[10](shortly M- F_β-continuos)if $f^{-1}(A)$ is F_β-open in X, for every F_β -open set A of Y).

(iii) Fuzzy irresolute [6] iff $f^{-1}(A)$ is a fuzzy semi-open subset of X, for each fuzzy semi open subset A of Y.

(iv) A fuzzy strongly continuous function [7] iff $f^{-1}(A)$ is fuzzy clopen in X, for every fuzzy subset A in Y.

DEFINITION 1.2 : A fuzzy point x_t is said to be quasi-coincident with a fuzzy set A in X if $t + A(x) > 1$. A fuzzy set A in X is said to be quasi-coincident with a fuzzy set B in X, denoted by AqB. If there exists a point x in X such that $A(x) + B(x) > 1$. [9].

LEMMA 1.1: Let $f : X \to Y$ be a mapping and x_t be a fuzzy point of X. Then

(i) $f(x_t)$ q B => x_t qf^{-1} (B), for every fuzzy set B of Y.

(ii) x_tq A => $f(x_t)_q$ f(A), for every fuzzy set A of X. [14].

2. STRONGLY $F_{s\beta}$ -IRRESOLUTE MAPPING:

In this section we introduce a new class of mapping, called strongly $F_{s\beta}$ -irresolute mapping, some examples and characterizations are also examined.

DEFINITION 2.1 : A mapping $f : X \to Y$ is said to be strongly fuzzy semi -β - irresolute (shortly st - $F_{s\beta}$-irresolute) if $f^{-1}(A)$ is Fs-open in X, for every F_β -open set A of Y. Equivalently we may say that f is st- $F_{s\beta}$-irresolute, iff $f^{-1}(A)$ is F_s-closed in X, for every F_β -closed set A of Y.

REMARK 2.1 : Every strongly $F_{\alpha p}$-irresolute mapping is st-$F_{s\beta}$-irresolute and every st-$F_{s\beta}$- irresolute mapping is M-Fβ-continuous but the converse may not be true in general. For,

EXAMPLE 2.1 : Let X = {a, b} and Y = {x, y}. Fuzzy sets A, B and H are defined as: A (a) = 0.5, A (b) = 0.5 ; B (x) = 0.6 , B(y) = 0.2 ; H(x) = 0.7, H(y) = 0.7; Let τ = {0, A, I} and σ = {0, B, I}. Then the mapping f : (X, Y) $\rightarrow$ (Y, σ) defined by f (a) = x and f (b) = y is st-$F_{s\beta}$-irresolute but not st-$F_{\alpha p}$- irresolute, because H is F_β-open in Y, but f^{-1}(H) is not Fα-open in X.

EXAMPLE 2.2 : In example 2.1, if we take A (a) = 0.4, A (b) = 0.4 then the mapping f : (X, τ) $\rightarrow$ (Y, σ) is M-fuzzy β-continuous but not st-$F_{s\beta}$-irresolute, because H is F_β-open in Y, but f^{-1} (H) is not F_s-open in X.

THEOREM 2.1 : For mapping f : X$\rightarrow$Y, the following are equivalent:

(a) f is st- $F_{s\beta}$ -irresolute;

(b) For every fuzzy point x_t of X and every Fβ -open set V of Y containing f(x_t) , there exists a Fs-open set U of X containing x_t st. f (U) $\leq$ V ;

(c) For every fuzzy point x_t of X and every F_β-open set V of Y containing f(x_t), there exists a Fs- open set U of X such that x $\in$ U $\leq f^{-1}$ (V) ;

(d) For every fuzzy point x_t of X, the inverse image of each fuzzy β-neighbourhood of f (x_t), is fuzzy semi-neighbourhood of x_t;

(e) For every fuzzy point x_t of X and each fuzzy β - neighbourhood E of f(x_t), there exists a fuzzy semi- neighbourhood A of x_t such that f(A) $\leq$ E;

(f) f^{-1}(V) $\leq$ Cl (Int (f^{-1}(V)) for every fuzzy β-open set V of Y;

(g) f^{-1}(H) is Fs- closed in X, For every Fβ-closed set H of Y;

(h) Int (Cl (f^{-1}(E))) $\leq$ f^{-1}(β Cl (E)), For every fuzzy subset E of Y;

(i) f (Int (Cl (A))) $\leq$ βCl (f (A)) for every fuzzy subset A of X.

PROOF : (a) <=> (b) <=> (c) ; (d) => (e) : Obvious.

(b) => (f) :Let V be any F_β -open set of Y and $x_t \in f^{-1}$ (V). By (b) there exists a Fs- open set U of X containing x_t such that f (U) $\leq$ V. Thus we have $x_t \in$U $\leq$ Cl (Int(U)) $\leq$ Cl (Int (f^{-1}(V))) and hence f^{-1}(V) $\leq$ Cl (Int (f^{-1} (V))).

(f) => (g) : Let H be any F_β-closed set of Y. Set V = Y – H, then V is F_β- open in Y. By (f) , we obtain f^{-1} (V) $\leq$ Cl (Int (f^{-1} (V))) and hence f^{-1} (H) = X – f^{-1}(Y – H) = X – f^{-1} (V) is Fs-closed in X.

(g) => (h) : Let E be any fuzzy set of Y. Since βCl (E) is F_β - Closed subset of Y, f^{-1} (βCl (E)) is Fs–Closed in X and hence Int (Cl (f^{-1}(βCl (E))))$\leq$ f^{-1} (βCl (E)). Therefore we obtain Int (Cl (f^{-1}(E))) $\leq$ f^{-1} (βCl (E)).

(h) => (i) : Let A be any fuzzy subset of X . By (h) we have Int (Cl (A) ≤ Int (Cl (f^{-1} (f (A)))) ≤ f^{-1} (βCl (f (A))) and hence f (Int (Cl (A))) ≤ βCl (f (A)).

(i) =>(a) : Let V be any F_β-open set of Y. since $f^{-1}(Y - V) = X - f^{-1}(V)$ is fuzzy subset of X and by (i), we obtain f (Int (Cl (f^{-1}(Y – V)))) ≤ βCl (f(f^{-1}(Y – V))) ≤ βCl (Y–V) = Y–βInt (V) = Y–V and hence X – (Cl (Int (f^{-1}(V))) = Int (Cl (X – f^{-1} (V))) = Int (Cl (f^{-1}(Y – V))) ≤ f^{-1} (f (Int (Cl (f^{-1}(Y – V))))) ≤ f^{-1} (Y – V) = X – f^{-1} (V). Therefore, we have f^{-1}(V) ≤ Cl (Int (f^{-1}(V))) and hence f^{-1} (V) is Fs-open in X. Thus, f is st - $F_{s\beta}$ -irresolute.

(a) => (d) Let x_t be fuzzy point in X and V be β- neighbourhood of f(x_t) then there exists a F_β-open set G in Y such that f(x_t) ∈ G ≤ V. Now f^{-1}(G) is Fs-open in X and x_t∈f^{-1} (G) ≤ f^{-1}(V). Thus f^{-1}(V) is fuzzy semi neighbourhood of x_t in X .

(e) => (b) Let x_t be a fuzzy point in X and V be any F_β-open in Y such that f(x_t)∈V. Then V is fuzzy β-nbd of f(x_t), so there exists a fuzzy semi neighbourhood A of x_t such that x_t∈ A and f(A) ≤ V. Hence there exist Fs-open set U of X such that x_t ∈U ≤ A and f (U) ≤ f(A) ≤ V.

THEOREM 2.2: For a mapping f : X→ Y following are equivalent;

(a) f is st- $F_{s\beta}$ - irresolute;

(b) For each fuzzy point x_t of X, and every F_β-open set E of Y such that f(x_t) q E, there exist Fs-open set A of X such that x_t q A and f(A) ≤ E ;

(c) For every fuzzy point x_t of X and every F_β -open set E of Y, such that , f(x_t) q E , there exist Fs open set A of X such that x_t q A and A ≤ f^{-1}(E);

PROOF : (a) => (b) : Let x_t be fuzzy point of X and E be F_β-open set of Y such that f(x_t) q E . Then f^{-1}(E) is Fs-open in X and x_t qf^{-1}(E), by Lemma 1.1. If we take A = f^{-1} (E) then x_t q A and A = f^{-1}(f (E)) ≤ E.

(b) => (c) : Let x_t be a fuzzy point in X and E be F_β-open in Y such that f(x_t)qE. Then by (b), there exists Fs-open set A in X such that x_t qA and f (A) ≤ E. Hence x_t q A and A ≤ f^{-1} (f (A)) ≤ f^{-1}(E).

(c) => (a) : Let E be any F_β-open in Y and x_t be a fuzzy point in X such that x_t ∈f^{-1}(E). Then f(x_t)∈E. Choose the fuzzy point x_t^c (x) =1 – x_t (x). Then f(x_t^c) q E and so by (c), there exist Fs-open set A of X such that (x_t^c) q A and f(A) ≤ E . Now (x_t^c)q A => x_t^c (x) + A (x) = l – x_t(x) + A(x) > 1. It follows that x_t ∈A. Thus x_t ∈ A ≤ f^{-1} (E) . Hence f^{-1}(E) is Fs-open in X.

LEMMA 2.1: Let g : X → X × Y be the graph of a mapping f : X →Y. If A is fuzzy set of X and B is fuzzy set of Y, then g^{-1} (A×B) = A ∩ f^{-1} (B) .[1].

THEOREM 2.3: $f : X \to Y$ be a mapping. If the graph mapping $g : X \to X \times Y$ of f is st -Fs_β-irresolute, then f is st-Fs_β -irresolute.

Proof : Let A be any F_β- open set of Y, then by lemma 2.1, $f^{-1}(A) = 1 \cap f^{-1}(A) = g^{-1}$ ($I \times A$). Since A is F_β open in Y, $1 \times A$ is F_β -open in $X \times Y$. Since g is st -Fs_β -irresolute, g^{-1} ($I \times A$) is -Fs open in X and consequently f is st-$F_{s\beta}$-irresolute.

3. COMPOSITIONS OF ST -$F_{s\beta}$-IRRESOLUTE MAPPINGS

In this section the composition of st-$F_{S\beta}$ irresolute mappings with other fuzzy mappings are studied.

THEOREM 3.1 : If $f : X \to Y$ is st - $F_{s\beta}$ -irresolute and $g : Y \to Z$ is M fuzzy β-continuous , then $gof : X \to Z$ is st - $F_{s\beta}$ -irresolute.

COROLLARY 3.1 : The composition of two st -$F_{s\beta}$ -irresolute mapping is st-$F_{s\beta}$ -irresolute.

COROLLARY 3.2 : If $f : X \to Y$ is fuzzy strongly continuous and $g: Y \to Z$ is st $F_{s\beta}$ -irresolute, then $gof : X \to Z$ is st - $F_{s\beta}$ -irresolute.

THEOREM 3.2: If $f : X \to Y$ is fuzzy irresolute and $g : Y \to Z$ is st-$F_{s\beta}$-irresolute, then $gof : X \to Z$ is st-$F_{s\beta}$- irresolute.

THEOREM 3.3 : Let P_i be projection function from ΠX_i onto X_i ,then if . $f : X \to \Pi x_i$ is st-$F_{s\beta}$-irresolute, so is P_i o f for each $i \in \wedge$.

PROOF: Let V_i be any F_β-open set of X_i. Since P_i is fuzzy continuous and fuzzy open, it is M-fuzzy β-continuous and hence $P_i^{-1}(V_i)$ is F_β-open in ΠX_i. Since f is st-$F_{s\beta}$ -irresolute, $f^{-1}(P_i^{-1}(V_i))=(P_i \circ f)^{-1}(V_i)$ is Fs-open in X . Hence is P_i o f st-$F_{s\beta}$-irresolute for each $i \in \wedge$.

REFERENCES

[1] Azad K. K. : On fuzzy semi continuity, Almost continuity and fuzzy weakly continuity; J. Math. Anal. Appl. , 82 (1981) 14-32.

[2] Balasubramanian G: Fuzzy β-open sets and fuzzy β-separation axioms; Kybernetica, 35 (1991) 215-223.

[3] Binshahna A. S. : On Fuzzy strongly semi continuity, and precontinuity; Fuzzy Sets and Systems, 14(1991) 303-308.

[4] Chang C. L. : Fuzzy Topological spaces; J. Math. Anal. Appl., 24 (182-90).

[5] Mashhour A. S., Ghamin M.H. and Fath Alla : On Fuzzy non continuous mapping; Bull. Calcutta. Math. Soc., 78(1986) 57-69.

[6] Mukherjee M. N. and Sinha S. P. , irresolute and almost open function betbeen fuzzy Topological spaces; Fuzzy Sets and Systems, 29 (1989) 381-388.

[7] Mukhergee M. N. and Ghosh B., some stronger Forms of Fuzzy continuous mapping on Fuzzy topological spaces, Fuzzy Sets and Systems 36(1990) 375-387.

[8] Prasad R. Thakur S. S. and Saraf R. K. : Fuzzy α irresolute mapping; J. Fuzzy Math. , 2(1994) 235-239.

[9] Pu. P.M and Liu. Y.M. : Fuzzy Topology I Neigh bourhood structure of fuzzy point and moor Smith convergence; J. Math. Anal App. , 76(1989) 571-594.

[10] Roja E. and Balasubrumanian G: On fuzzy β $T_{½}$ spaces and its generalization, Bull. Calcutta Math. Soc., 94(2002)413-420.

[11] Saraf R. K., Navlgi G. and Caldas M. : On Fuzzy α Preirresolute mapping; Advances in Fuzzy Math. 3(2008)19-25.

[12] Singal M.K. and Prakash N : Fuzzy Preopen sets and fuzzy pre separation axioms; Fuzzy Sets and Systems 44(1991), 271-281.

[13] Thakur S. S. and Singh S. : On Fuzzy semi Preopen Sets and fuzzy semi pre Contínity, Fuzzy Sets and Systems, (98)283-391.

[14] Yalvac T.H.: Fuzzy sets and function on Fuzzy spaces; J. Math. Anal. Appl., 126(1987) 409-423.

[15] Zadeh L.A. : Fuzzy sets; Inform and control, 8(1965) 338-353.