1. ON STRONGLY $F_{\alpha^{+}}$-IRRESOLUTE MAPPINGS

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ABSTRACT: The aim of this paper is to introduce a new class of mappings called strongly $F_{\alpha^{+}}$- irresolute mapping, its characteristic properties, examples and composition with other mappings are studied.

Keywords: $F_{\alpha}$-open, $F_{\alpha}$-open, fuzzy topological space, st-$F_{\alpha}$- irresolute mapping.


1. INTRODUCTION

In this paper, we introduce a new class of mapping called st-$F_{\alpha}$ - irresolute mapping. In section 2 some examples and characterizations of new mappings are examined. In section 3 the composition of st-$F_{\alpha}$ - irresolute mapping with other fuzzy mappings are studied.

The concept fuzzy has invaded almost all branches of mathematics, since the introduction of the concept of fuzzy sets by Zadeh [15]. The theory of fuzzy topological spaces was introduced and developed by Chang [4] and since then various notions in classical topology has been extended to fuzzy topological spaces. Our motivation in this paper is to define strongly $F_{\alpha^{+}}$- irresolute mappings and investigate its properties. The newly defined class of mapping is stronger then M-fuzzy $\beta$-continuous mapping and is a generalization of strongly $F_{\alpha}$- irresolute mapping.

Throughout this note, spaces, always mean fuzzy topological spaces and $f : X \rightarrow Y$ denotes a mapping of a space $X$ into a space $Y$. Let $A$ be a fuzzy subset of a space $X$. The closure and the interior of a fuzzy set $A$ are denoted by $Cl (A)$ and $Int (A)$ respectively. The notation and terminologies not explained in this paper may be found in [6]. Definitions and results which will be needed in this paper, are recalled here:

DEFINITION 1.1 Let $A$ be a fuzzy subset in a space $X$

(a) $A$ is called :

(i) Fuzzy $\alpha$-open [3] (shortly $F_{\alpha}$-open) iff $A \subseteq \text{Int} (\text{Cl} (\text{Int} (A)))$.

(ii) Fuzzy semi open [1] (shortly $F_{s}$-open) iff $A \subseteq \text{Cl} (\text{Int} (A))$.

(iii) Fuzzy pre-open [3] (shortly $F_{p}$-open) iff $A \subseteq \text{Int} (\text{Cl} (A))$.

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(iv) Fuzzy $\beta$-open [5] (shortly $F_{p}$-open) iff $A \subseteq \text{Cl} (\text{Int} (\text{Cl} (A)))$.

The complement of these sets are respectively called $F_{c}$-closed (resp. $F_{c}$-closed, $F_{p}$-closed).

(b) The fuzzy pre-interior [12] (resp. fuzzy $\beta$-interior [2]) of $A$, denoted by

$\text{pint}(A)$ (resp. $\text{pint}(A)$) is the union of all $F_{p}$-open (resp. $F_{c}$-open) subsets contained in $A$.

(ii) The fuzzy pre closure[12] (resp. fuzzy $\beta$-closure [2]) of $A$, denoted by $p\text{Cl}$

$\text{A}$ (resp. $[\text{Cl}(A)$) is the intersection of all $F_{p}$-Closed (resp. $F_{c}$-closed) subsets containing $A$.

(c) Let $f : X \to Y$ be mapping. Then $f$ is called:

(i) Strongly fuzzy $\alpha$ pre irresolute [11] (shortly st- $F_{p}$-$\alpha$ irresolute) if $f^{-1}(A)$ is $F_{p}$-open in $X$, for every $F_{p}$-open set $A$ of $Y$.

(ii) M-fuzzy $\beta$-continuous[10](shortly M- $F_{p}$-continuous)if $f^{-1}(A)$ is $F_{p}$-open in $X$, for every $F_{p}$-open set $A$ of $Y$.

(iii) Fuzzy irresolute [6] iff $f^{-1}(A)$ is a fuzzy semi-open subset of $X$, for each fuzzy semi-open subset $A$ of $Y$.

(iv) A fuzzy strongly continuous function [7] iff $f^{-1}(A)$ is fuzzy clopen in $X$, for every fuzzy subset $A$ in $Y$.

DEFINITION 1.2: A fuzzy point $x_{i}$ is said to be quasi-coincident with a fuzzy set $A$ in $X$ if $f + A (x) > 1$. A fuzzy set $A$ in $X$ is said to be quasi-coincident with a fuzzy set $B$ in $X$, denoted by $A \# B$. If there exists a point $x$ in $X$ such that $A (x) = B (x) > 1$ [9].

LEMMA 1.1: Let $f : X \to Y$ be a mapping and $x_{i}$ be a fuzzy point of $X$. Then

(i) $f(x_{i}) \# B \Rightarrow x_{i}, f^{-1}(B)$, for every fuzzy set $B$ of $Y$.

(ii) $x_{i} \# A \Rightarrow f(x_{i}) \# A$, for every fuzzy set $A$ of $X$. [14].

2. STRONGLY $F_{s}$-IRRESOLUTE MAPPING:

In this section we introduce a new class of mapping, called strongly $F_{s}$-$\alpha$ irresolute mapping, some examples and characterizations are also examined.

DEFINITION 2.1: A mapping $f : X \to Y$ is said to be strongly fuzzy semi-$\beta$- irresolute (shortly st- $F_{s}$-irresolute) if $f^{-1}(A)$ is $F_{s}$-open in $X$, for every $F_{s}$-open set $A$ of $Y$.

Equivalently we may say that $f$ is st- $F_{s}$-$\alpha$ irresolute, iff $f^{-1}(A)$ is $F_{s}$-closed in $X$, for every $F_{s}$-closed set $A$ of $Y$. 

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REMARK 2.1: Every strongly $F_{st}$- irresolute mapping is st-$F_{st}$- irresolute and every st-$F_{st}$- irresolute mapping is $M$-$\beta$-continuous but the converse may not be true in general. For,

EXAMPLE 2.1: Let $X = \{a, b\}$ and $Y = \{x, y\}$. Fuzzy sets $A, B$ and $H$ are defined as:

$A(a) = 0.5, A(b) = 0.5; B(x) = 0.6, B(y) = 0.2; H(x) = 0.7, H(y) = 0.7$; Let $\beta = \{0, A, B\}$ and $\alpha = \{0, B, I\}$. Then the mapping $f: (X, \beta) \rightarrow (Y, \alpha)$ defined by $f(a) = x$ and $f(b) = y$ is st-$F_{st}$- irresolute but not st-$F_{st}$- irresolute, because $H$ is $F_{st}$-open in $Y$, but $f^{-1}(H)$ is not $F_{st}$-open in $X$.

EXAMPLE 2.2: In example 2.1, if we take $A(a) = 0.4, A(b) = 0.4$ then the mapping $f: (X, \beta) \rightarrow (Y, \alpha)$ is $M$-$\mu$-continuous but not st-$F_{st}$-irresolute, because $H$ is $F_{st}$-open in $Y$, but $f^{-1}(H)$ is not $F_{st}$-open in $X$.

THEOREM 2.1: For mapping $f: X \rightarrow Y$, the following are equivalent:

(a) $f$ is st-$F_{st}$- irresolute;

(b) For every fuzzy point $x_i$ of $X$ and every $F_{\beta}$-open set $V$ of $Y$ containing $f(x_i)$, there exists a $F_{\alpha}$-open set $U$ of $X$ containing $x_i$ such that $f(U) \subseteq V$;

(c) For every fuzzy point $x_i$ of $X$ and every $F_{\beta}$-open set $V$ of $Y$ containing $f(x_i)$, there exists a $F_{\alpha}$-open set $U$ of $X$ such that $x_i \in U \subseteq f^{-1}(V)$;

(d) For every fuzzy point $x_i$ of $X$, the inverse image of each fuzzy $\beta$-neighbourhood of $f(x_i)$ is fuzzy semi-neighbourhood of $x_i$;

(e) For every fuzzy point $x_i$ of $X$ and each fuzzy $\beta$-neighbourhood $E$ of $f(x_i)$, there exists a fuzzy semi-neighbourhood $A$ of $x_i$ such that $f(A) \subseteq E$;

(f) $f^{-1}(V) \subseteq Cl(Int(f^{-1}(V)))$ for every fuzzy $\beta$-open set $V$ of $Y$;

(g) $f^{-1}(H)$ is $F_{\alpha}$-closed in $X$. For every $F_{\beta}$-closed set $H$ of $Y$;

(h) $Int(Cl(f^{-1}(E))) \subseteq f^{-1}(Int(Cl(E)))$. For every fuzzy subset $E$ of $Y$;

(i) $Int(Cl(f^{-1}(\alpha))) \subseteq f^{-1}(Int(Cl(\beta)))$ for every fuzzy subset $\alpha$ of $X$.

PROOF: (a) $\iff$ (b) $\iff$ (c) $\iff$ (d) $\iff$ (e) Obvious.

(b) $\Rightarrow$ (f): Let $V$ be any $F_{\beta}$-open set of $Y$ and $x_i \in f^{-1}(V)$. By (b) there exists a $F_{\alpha}$-open set $U$ of $X$ containing $x_i$ such that $f(U) \subseteq V$. Thus we have $x_i \in U \subseteq Cl(Int(U)) \subseteq Cl(Int(f^{-1}(V)))$ and hence $f^{-1}(V) \subseteq Cl(Int(f^{-1}(V)))$.

(f) $\Rightarrow$ (g): Let $H$ be any $F_{\beta}$-closed set of $Y$. Set $V = Y - H$, then $V$ is $F_{\beta}$-open in $Y$. By (f), we obtain $f^{-1}(V) \subseteq Cl(Int(f^{-1}(V)))$ and hence $f^{-1}(H) = X - f^{-1}(Y - H) = X - f^{-1}(V)$ is $F_{\alpha}$-closed in $X$.

(g) $\Rightarrow$ (h): Let $E$ be any fuzzy set of $Y$. Since $f^{-1}(E)$ is $F_{\beta}$-closed subset of $Y$, $f^{-1}(Int(Cl(f^{-1}(E)))) \subseteq f^{-1}(Int(Cl(E)))$. Therefore we obtain $Int(Cl(f^{-1}(E))) \subseteq f^{-1}(Int(Cl(E)))$.
(h) $\Rightarrow$ (i): Let $A$ be any fuzzy subset of $X$. By (h) we have $\text{int} \left( \text{Cl} (A) \right) \subseteq \text{int} \left( \text{Cl} (f^{-1} (f(A))) \right) \subseteq f^{-1} \left( \text{Cl} (f(A)) \right)$ and hence $f (\text{int} \left( \text{Cl} (A) \right)) \subseteq \text{Cl} (f(A))$.

(i) $\Rightarrow$ (a): Let $V$ be any $F_2$-open set of $Y$. Since $f^{-1}(Y - V) = X - f^{-1}(V)$ is fuzzy subset of $X$ and by (i), we obtain $f (\text{int} \left( \text{Cl} (f^{-1}(Y - V)) \right)) \subseteq \text{Cl} (f(f^{-1}(Y - V))) \subseteq \text{Cl} (Y - V)$ and hence $X - \text{Cl} (f^{-1}(Y - V)) = f^{-1}(Y - V)$. Therefore, we have $f^{-1}(Y - V) \subseteq \text{Cl} (f(f^{-1}(Y - V)))$ and hence $f^{-1}(V)$ is $F_2$-open in $X$. Thus, $f$ is at least $F_{20}$- irresolute.

(a) $\Rightarrow$ (d): Let $x_i$ be fuzzy point in $X$ and $V$ be $\beta$-neighbourhood of $f(x_i)$ then there exists a $F_2$-open set $G$ in $Y$ such that $f(x_i) \in G \subseteq V$. Now $f^{-1}(G)$ is $F_2$-open in $X$ and $x_i \in f^{-1}(G) \subseteq f^{-1}(V)$. Thus $f^{-1}(V)$ is fuzzy semi-neighbourhood of $x_i$ in $X$.

(b) $\Rightarrow$ (b): Let $x_i$ be a fuzzy point in $X$ and $V$ be any $F_2$-open in $Y$ such that $f(x_i) \in V$. Then $V$ is fuzzy $\beta$-nebd of $f(x_i)$, so there exists a fuzzy semi-neighbourhood $A$ of $x_i$ such that $x_i \in A$ and $f(A) \subseteq V$. Hence there exists $F_2$-open set $U$ of $X$ such that $x_i \in U \subseteq A$ and $f(U) \subseteq f(A) \subseteq V$.

**THEOREM 2.2:** For a mapping $f: X \rightarrow Y$ following are equivalent:

(a) $f$ is at least $F_{20}$- irresolute;

(b) For each fuzzy point $x_i$ of $X$, and every $F_2$-open set $E$ of $Y$ such that $f(x_i) \in E$, there exists $F_2$-open set $A$ of $X$ such that $x_i \in A$ and $f(A) \subseteq E$;

(c) For every fuzzy point $x_i$ of $X$ and every $F_2$-open set $E$ of $Y$, such that $f(x_i) \in E$, there exists $F_2$-open set $A$ of $X$ such that $x_i \in A$ and $f(A) \subseteq E$.

**PROOF:** (a) $\Rightarrow$ (b): Let $x_i$ be fuzzy point of $X$ and $E$ be $F_2$-open set of $Y$ such that $f(x_i) \in E$. Then $f^{-1}(E)$ is $F_2$-open in $X$ and $x_i \in f^{-1}(E)$, by Lemma 1.1. If we take $A = f^{-1}(E)$ then $x_i \in A$ and $f(A) \subseteq E$.

(b) $\Rightarrow$ (c): Let $x_i$ be a fuzzy point in $X$ and $E$ be $F_2$-open in $Y$ such that $f(x_i) \in E$. Then by (b), there exists $F_2$-open set $A$ in $X$ such that $x_i \in A$ and $f(A) \subseteq E$. Hence $x_i \in A$ and $f(A) \subseteq E$.

(c) $\Rightarrow$ (a): Let $E$ be any $F_2$-open in $Y$ and $x_i$ be a fuzzy point in $X$ such that $x_i \in f^{-1}(E)$. Then $f(x_i) \in E$. Choose the fuzzy point $x_i^* \in (x_i \in f^{-1}(E))$ and so by (c), there exist $F_2$-open set $A$ of $X$ such that $(x_i^*) \in A$ and $f(A) \subseteq E$. Now $(x_i^*) \subseteq A \Rightarrow x_i^* \in A$. Thus $x_i \in f^{-1}(E)$. Hence $f^{-1}(E)$ is $F_2$-open in $X$.

**LEMMA 2.1:** Let $g: X \rightarrow X \times Y$ be the graph of a mapping $f: X \rightarrow Y$. If $A$ is fuzzy set of $X$ and $B$ is fuzzy set of $Y$, then $g^{-1}(A \times B) = A \times f^{-1}(B)$ [1].
THEOREM 2.3: If: \( f: X \to Y \) be a mapping. If the graph mapping \( g: X \to X \times Y \) of \( f \) is st\( -\text{F}_{\alpha \beta} \)- irresolute, then \( f \) is st\( -\text{F}_{\alpha \beta} \)- irresolute.

Proof: Let \( A \) be any \( \text{F}_{\alpha \beta} \)-open set of \( Y \), then by lemma 2.1, \( f^{-1}(A) = 1 \cap f^{-1}(A) \cap g^{-1}(I \times A) \). Since \( A \) is \( \text{F}_{\alpha \beta} \)-open in \( Y \), \( I \times A \) is \( \text{F}_{\alpha \beta} \)-open in \( X \times Y \). Since \( g \) is st\( -\text{F}_{\alpha \beta} \)- irresolute, \( g^{-1}(I \times A) \) is \( \text{F}_{\alpha \beta} \)-open in \( X \) and consequently \( f \) is st\( -\text{F}_{\alpha \beta} \)- irresolute.

3. COMPOSITIONS OF ST\( -\text{F}_{\alpha \beta} \)-IRRESOLUTE MAPPINGS

In this section the composition of st\( -\text{F}_{\alpha \beta} \)- irresolute mappings with other fuzzy mappings are studied.

THEOREM 3.1: If \( f: X \to Y \) is st\( -\text{F}_{\alpha \beta} \)-irresolute and \( g: Y \to Z \) is M fuzzy \( \beta \)-continuous, then \( \text{gof}: X \to Z \) is st\( -\text{F}_{\alpha \beta} \)-irresolute.

COROLLARY 3.1: The composition of two st\( -\text{F}_{\alpha \beta} \)-irresolute mappings is st\( -\text{F}_{\alpha \beta} \)-irresolute.

COROLLARY 3.2: If \( f: X \to Y \) is fuzzy strongly continuous and \( g: Y \to Z \) is st\( -\text{F}_{\alpha \beta} \)- irresolute, then \( \text{gof}: X \to Z \) is st\( -\text{F}_{\alpha \beta} \)- irreolute.

THEOREM 3.2: If \( f: X \to Y \) is fuzzy irresolute and \( g: Y \to Z \) is st\( -\text{F}_{\alpha \beta} \)- irresolute, then \( \text{gof}: X \to Z \) is st\( -\text{F}_{\alpha \beta} \)- irresolute.

THEOREM 3.3: Let \( P_i \) be projection function from \( I \times X \), onto \( X \), then if \( f: X \to I \times X \) is st\( -\text{F}_{\alpha \beta} \)- irresolute, then \( P_i f \) is st\( -\text{F}_{\alpha \beta} \)- irresolute for each \( i \in I \).

PROOF: Let \( V_i \) be any \( \text{F}_{\alpha \beta} \)-open set of \( X \). Since \( P_i \) is fuzzy continuous and fuzzy open, it is M-fuzzy \( \beta \)-continuous and hence \( P_i^{-1}(V_i) \) is \( \text{F}_{\alpha \beta} \)-open in \( I \times X \). Since \( f \) is st\( -\text{F}_{\alpha \beta} \)- irresolute, \( f^{-1}(P_i^{-1}(V_i)) = (P_i f)^{-1}(V_i) \) is \( \text{F}_{\alpha \beta} \)-open in \( X \). Hence \( P_i f \) is st\( -\text{F}_{\alpha \beta} \)- irresolute for each \( i \in I \).

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