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MATHEMATICS

Pre - θ - perfect mappings and p - closed spaces

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Abstract. In this paper, we introduce Per - θ - perfect mappings and investigate some of their characterizations and properties . Also we give a characterization of p - closed spaces

Key words : Pre - θ - closed sets , filter base , p - closed spaces .

1. Introduction

A mapping $f : X \rightarrow Y$ is called perfect if f is closed and f^{-1} (y) is compact , for each y $\in$ Y . Whyburn [9] proved that a mapping $f : X \rightarrow Y$ is perfect if and only if for every filter base Φ on f (X) converging to y $\in$ Y , f^{-1} (Φ) is directed towards f^{-1} (y) . The purpose of the present paper is to introduce pre - θ - perfect mappings defined in a way similar to the above characterization of a perfect mapping and investigate some of their properties and characterizations . Also we give a characterization of p - closed spaces .

2. Preliminaries

Recall that a subset A of a space X is called preopen [4] if A $\subset$ int (cl (A)).The complement of a preopen set is called precolsed . The intersection of all preclosed sets containing A is called the preclosure of A and denoted by pcl (A) .

Definition 2.1[5] . Let A be a subset of a space X .

(i) A piont $x \in$ X is called a pre - θ - cluster point of A if pcl (V) $\bigcap$ A $\neq \phi$, for every preopen set V containing x .

(ii) The set of all pre- θ - cluster points of A is called the pre - θ - closure of A and is denoted by pcl_{θ} (A) .

(iii) A subset A of a space X is called pre - θ - closed if pcl_{θ} (A) = A .

(iv) The complement of a pre - θ - closed set is called pre - θ - open .

Remark 2.1. It is obvious that a pre - θ - open (resp. pre - θ - closed) set is preopen (resp. preclosed) , but the converse need not be true as shown by Example 3.3 of [2] .

Definition 2.2. Let X be a topological space .

(i) Apoint $x \in X$ is called a pre - θ - cluster point of a filter base Φ in X if $x \in \bigcap \{ pcl_\theta(F) : F \in \Phi \} = [ad]_{P\theta}(\Phi)$.

(ii) A filter base Φ in X $p\theta^*$ - converges [2] to a point $x \in X$ if for each preopen set A containing x, there exists an $F \in \Phi$ such that $F \subset pcl(A)$.

(iii) A filter base Γ is said to be subordinate [6] to a filter base Φ if for each $F \in \Phi$, there exists $G \in \Gamma$ such that $G \subset F$.

(iv) A filter base Φ is said to be $p\theta$ - directed towards $A \subset X$ if every filter base subordinate to Φ has a pre - θ - cluster point in A.

3. Pre - θ - perfect mappings.

Definition 3.1. A mapping $f : X \to Y$ is called pre - θ - perfect ($p\theta$ - perfect in short) if for every filter base Φ in $f(X)$ $p\theta^*$ - converging to $y \in Y$, $f^{-1}(\Phi)$ is $p\theta$ - directed towards $f^{-1}(y)$.

Remark 3.1. Continuity is not assumed on $p\theta$ - perfect mappings.

Definintion 3.2. A mapping $f : X \to Y$ is called pre - θ - closed ($p\theta$ - closed in short) if $pcl_\theta(f(A)) \subset f(pcl_\theta(A))$, for every subset A of X.

Theorem 3.1. *Every $p\theta$ - perfect mapping is $p\theta$ - closed.*

Proof. Suppose that $f : X \to Y$ is $p\theta$ - perfect mapping. Let A be any subset of X and $y \in pcl_\theta(f(A))$. Then there exists a filter base Φ on $f(A)$, $p\theta^*$ - converging to y. Put $\Gamma = \{ f^{-1}(F) \bigcap A : F \in \Phi \}$. Then Γ is a filter base in X and subordinate to the filter base $f^{-1}(\Phi)$. Since $f^{-1}(\Phi)$ is $p\theta$ - directed towards $f^{-1}(y)$, we have $f^{-1}(y) \bigcap ([ad]_{P\theta}(\Gamma)) \neq \phi$. Therefore, we obtain $y \in f(pcl_\theta(A))$. This implies that f is $p\theta$ - closed.

Theorem 3.2. *A mapping $f : X \to Y$ is $p\theta$ - closed if and only if the image $f(A)$ of each pre - θ - closed subset A of X is pre - θ - closed subset of Y.*

Proof. Obvious.

Theorem 3.3. *The composition $gof : X \to Y$ of $p\theta$-closed mappings $f : X \to Y$ and $g : Y \to Z$ is $p\theta$ - closed mapping.*

Proof. Obvious.

Theorem 3.4. *A mapping $f : X \to Y$ is $p\theta$-perfect if and only if $[ad]_{P\theta} f(\Phi) \subset f([ad]_{P\theta} \Phi)$, for every filter base Φ in X.*

Proof. Suppose $f : X \to Y$ is pθ-perfect mapping. Let Φ be a filter base in X and $y \in ([ad]_{P\theta} f(\Phi))$. Then there exists a filter base Γ in $f(X)$ which is subordinate to $f(\Phi)$ and $p\theta^*$ - converges to y . Put

$$H = \{ f^{-1}(G) \bigcap F : G \in \Gamma , F \in \Phi \},$$

then H is a filter base in X subordinate to $f^{-1}(\Gamma)$ .Since f is pθ - perfect , $f^{-1}(\Gamma)$ is pθ - directed towards $f^{-1}(y)$. Therefore , we have $f^{-1}(y) \bigcap ([ad]_{P\theta} H) \neq \phi$ and hence $y \in f([ad]_{P\theta} \Phi)$.

Conversely suppose that the condition holds and f is not pθ - perfect . Then there exists a filter base Φ in $f(X)$ such that Φ pθ^* - converges to a point $y \in Y$ and $f^{-1}(\Phi)$ is not pθ - directed towards $f^{-1}(y)$ .Thus there exists a filter base Γ in X which is subordinate to $f^{-1}(\Phi)$ and $f^{-1}(y) \bigcap ([ad]_{P\theta} \Gamma) = \phi$. Therefore , we have $y \notin ([ad]_{P\theta} f(\Gamma))$ and hence $y \notin pcl_\theta(f(G_1))$ for some $G_1 \in \Gamma$. Then there exists a preopen set V containing y such that $(pcl(V)) \bigcap f(\Gamma_1) = \phi$. Since Φ pθ^* - converges to y and Γ is subordinate to $f^{-1}(\Phi)$, there exists $G_2 \in \Gamma$ such that $f(G_2) \subset pcl(V)$. Consequently , we have $G_1 \bigcap G_2 = \phi$. This contradicts that Γ is a filter base .

4. p – closed spaces

Definition 4.1. A space X is called p – closed [3] if every cover of X by preopen sets has a finite subcover whose preclosures cover X .

Definition 4.2. A subset A of a space X is called p – closed relative to X if for every cover $\{ V_\alpha : \alpha \in \Delta \}$ of A by preopen sets of X , there exists a finite subfamily Δ_O of Δ such that $A \subset \bigcup \{ pcl(V_\alpha) : \alpha \in \Delta_O \}$.

Theorem 4.1. *A subset B of a space X is p – closed relaive to X if and only if $B \bigcap ([ad]_{P\theta} \Phi) \neq \phi$, for every filter base Φ in B .*

Proof. Suppose that B is p – closed relative to X . Assume that there exists a filter base Φ in B such that $B \bigcap ([ad]_{P\theta} \Phi) = \phi$. Then for each $x \in B$ there exists a preopen set V_x containing x and an $F_x \in \Phi$ such that $F_x \bigcap pcl_\theta(V_x) = \phi$. Since B is p – closed relative to X , there exists a finite number of points $x_1, x_2, \ldots, x_n$ in B such that

$$B \subset \bigcup \{ pcl(V_{x_i}) : i = 1,2,\ldots, n \}.$$

Put $F = \bigcap \{ F_{x_i} : i = 1,2,\ldots, n \}$, then we obtain $F \bigcap B = \phi$. This contradicts that Φ is a filter base .

Conversely suppose B $\bigcap$([ad $]_{P\theta}$ Φ) $\neq \phi$, for every filter base Φ in B . Assume that B is not p – closed relative to X . Then there exists a cover { V_α : $\alpha \in \Delta$ } of B by preopen sets of X such that .

$$B \not\subset \bigcup \{ \text{pcl}\,(V_\alpha) : \alpha \in \nabla \}, \text{ for every } \nabla \in \Gamma(\Delta),$$

where $\Gamma(\Delta)$ denotes the family of all finite subsets of Δ . Now put

$$F_\nabla = \bigcap \{ B - \text{pcl}\,(V_\alpha) : \alpha \in \nabla \}, \text{ for each } \nabla \in \Gamma(\Delta).$$

Then Φ = { F_∇ : $\nabla \in \Gamma(\Delta)$} is a filter base in B and B $\bigcap$([ad $]_{P\theta}$ Φ) = ϕ . This is a contradiction . Therefore B is p – closed relative to X

Theorem 4.2. *If a mapping f : X → Y is pθ - perfect , then f^{-1} (B) is p – closed relative to X , for every p – closed relative to Y set B of Y .*

Proof. This follows from Theorem 3.4. and Theorem 4.1.

Theorem 4.3. *A mapping f : X → Y is pθ - perfect if and only if*
(i) f is pθ - closed , and
(ii) f^{-1} (y) is p – closed relative to X , for each y ∈ Y .

Proof. Necessity. Follows from Theorem 3.1 and Theorem 4.2.

Sufficiency. This is proven in a similar manner as the proof of conversely of Theorem 3.4.

Theorem 4.4. *The composition gof : X → Z of pθ-perfect mappings f : X → Y and g : Y → Z is pθ - perfect mapping .*

Proof. For pθ - closedness of (gof) , this follows from Theorem 3.1 and Theorem 3.3 and $(gof)^{-1}$ (z) = f^{-1} (g^{-1} (z)) is p–closed follows from Theorem 4.3 and Theorem 4.2.

Theorem 4.5. *Let S be a singleton with its unique topology . For a space X , the following statements are equivalent*
(i) X is p – closed
(ii) The constant mapping c : X → S is pθ - perfect .

Proof. The equivalence (i) ⇔ (ii) follows from Theorem 4.3.

Theorem 4.6. *If f : X → Y is pθ - perfect mapping and Y is p – closed , then X is p – closed .*

Proof. We show that the constant mapping k : X → S is pθ - perfect . Since Y is p – closed , therefore the mapping c : Y → S is pθ - perfect . Now k is pθ - perfect follows by noting that it is the composition (cof) of two pθ - perfect mappings . Hence X is p – closed .

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الرواسم قبل التامة من النوع θ وفراغات p المغلقة

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في هذا البحث قدمنا مفهوم الرواسم قبل التامة من النوع θ وقمنا بمناقشة بعض من خواصها وخصائصها , كما قمنا أيضاً في هذا البحث بتقديم بعض الخصائص لفراغات p المغلقة .